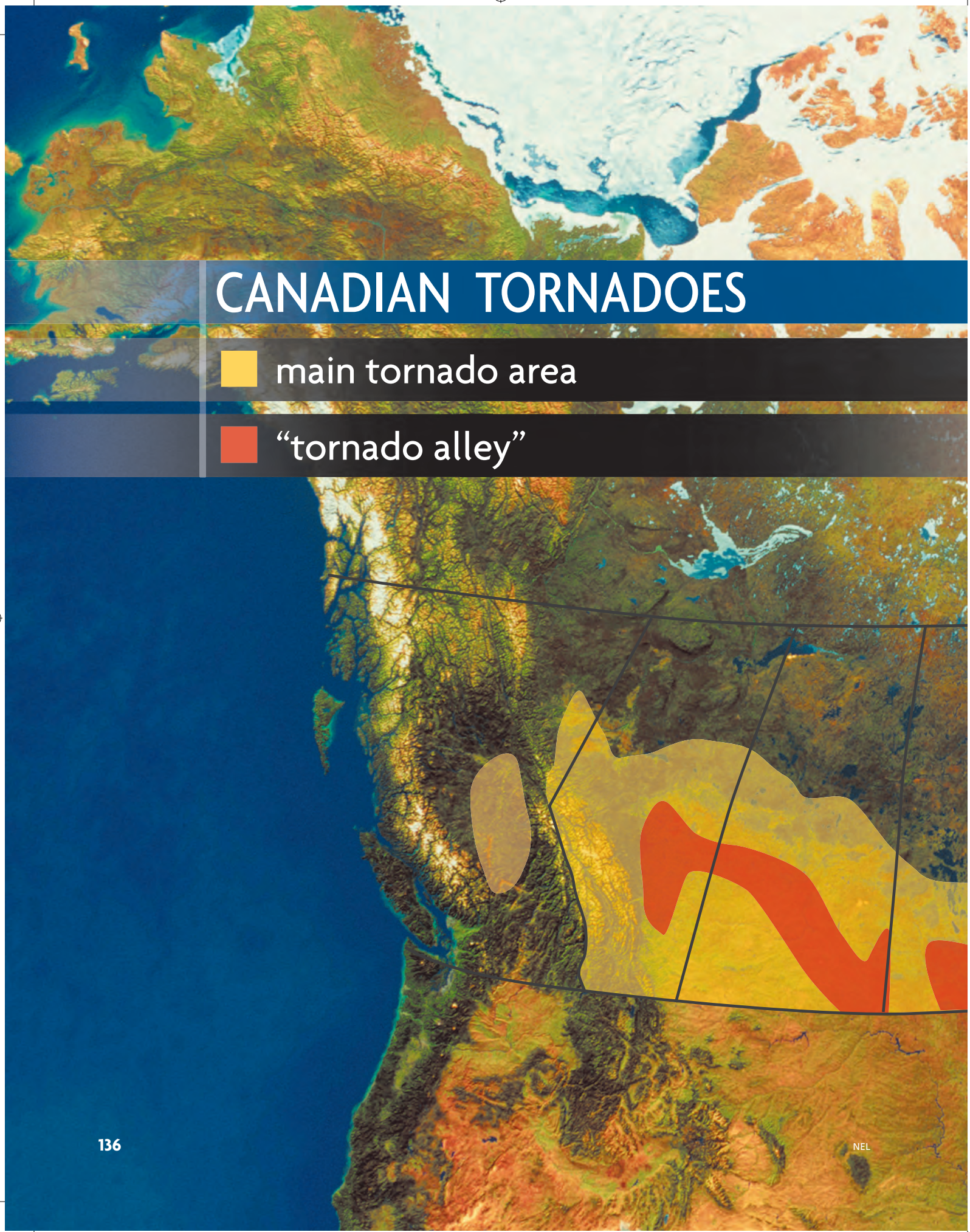

NEL

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Chapter

3

Probability

LEARNING GOALS

You will be able to develop your critical thinking skills related to uncertainty by

- Determining the probability and odds that an event will or will not occur
- Solving problems that involve dependent and independent events
- Solving problems that involve permutations and combinations
- Solving problems that involve mutually exclusive and non-mutually exclusive events

? Probability is used in many situations, such as predicting weather. How would you react if there was a 60% chance of rain? Would you react differently if there was a 60% chance of a tornado? Explain.

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3 Getting Started

YOU WILL NEED

- 2 dice
- graph paper

Dice Differences

Jack and Jill are playing a game. They roll two dice and determine the difference between the numbers rolled.

? How many differences are possible?

A. Predict the following:
- Which difference will occur most often?
- Which difference will occur least often?
- Which differences will occur equally often?

B. With a partner, roll two dice 36 times. Record the difference each time as a positive value. How closely do your results match your predictions?

C. Represent your results using a bar graph.

D. Determine the **experimental probability** for each difference as a fraction.

E. Compare your experimental results with another group's experimental results. Were they similar? Combine the two sets of results, and repeat part D for the combined results. How do these probabilities compare with those from part D?

F. Combine the results of all the groups in your class. Determine the experimental probability for these combined results. Express each probability as a decimal.

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G. Copy and complete this outcome table. Record the differences as positive values.

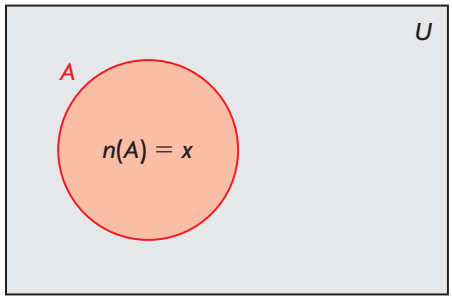
Differences in Rolls of Two Dice						
Die 1/Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4	3					
5						
6						

H. Determine the **theoretical probability** of each difference as a fraction. How closely does the experimental probability for the class results match the theoretical probability?

WHAT DO You Think?

Decide whether you agree or disagree with each statement. Explain your decision.

1. The probability that event X will happen is $\frac{2}{9}$. The probability that event Y will happen is $\frac{5}{9}$. Therefore, the probability that either event X or event Y will happen is $\frac{(2 + 5)}{9}$ or $\frac{7}{9}$.
2. Ideas from set theory are useful when you are solving probability problems.
3. You can use a Venn diagram to determine the probability of an event.



4. You can solve probability problems using the same techniques you use to solve counting problems. For example, suppose that you are going to draw two balls from a bag with five different-coloured balls. You can use combinations to determine the probability that you will draw a red ball and a blue ball.

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3.1 Exploring Probability

YOU WILL NEED

- slips of paper
- 2 containers

GOAL

Use probability to make predictions.

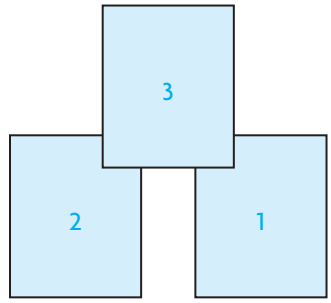
EXPLORE the Math

Sasha and Mika have to invent a **fair game** for a class project. Sasha suggests this game:

fair game

A game in which all the players are equally likely to win; for example, tossing a coin to get heads or tails is a fair game.

- Two people play.
- Both players put three identical slips of paper, numbered 1, 2, and 3, into their own container.



- For each turn, both players draw one slip of paper from their container.
  - Player 1 scores a point if the product of the two numbers drawn is less than their sum.
  - Player 2 scores a point if the product of the two numbers drawn is greater than their sum.
  - Neither player gets a point if the product and sum are equal.
- After each turn, the players return their slip of paper to their container.
- A game consists of 10 turns.

? Is Sasha's game a fair game?

Reflecting

A. Is playing Sasha's game once a good way to decide if it is fair? Explain.

B. Mika says that you can use theoretical probability to determine if the game is fair. Do you agree? Explain.

C. When you play Sasha's game, would you rather be player 1 or player 2? Explain.

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In Summary

Key Ideas

- Knowing the probability of an event is useful when making decisions.
- The **experimental probability** of event A is represented as

$$P(A) = \frac{n(A)}{n(T)}$$

where $n(A)$ is the number of times event A occurred and $n(T)$ is the total number of trials, T, in the experiment.

- The **theoretical probability** of event A is represented as

$$P(A) = \frac{n(A)}{n(S)}$$

where $n(A)$ is the number of favourable outcomes for event A and $n(S)$ is the total number of outcomes in the sample space, S, where all outcomes are equally likely.

- A game is fair when all the players are equally likely to win.

Need to Know

- An event is a collection of outcomes that satisfy a specific condition. For example, when throwing a regular die, the event "throw an odd number" is a collection of the outcomes 1, 3, and 5.
- The probability of an event can range from 0 (impossible) to 1 (certain). You can express probability as a fraction, a decimal, or a percent.
- You can use theoretical probability to determine the likelihood that an event will happen.

FURTHER Your Understanding

1. How can you change Sasha's game so that it is fair? Explain.

2. Consider each game below. Is it fair? If it is not fair, which player has the advantage? Explain.

a) Matt and Pat each toss a coin. If the coins land as both heads or both tails, Matt wins. If the coins land as a head and a tail, Pat wins.

b) Treena, Lena, and Gina each toss a coin. If all three coins land as heads, Treena wins. If all three coins land as tails, Lena wins. Otherwise, Gina wins.

c) Ann and Dan each roll a die. If the sum of the two dice is greater than 7, Ann wins. If the sum is less than 7, Dan wins. If the sum is 7, they tie.

3. Everard says he has a 120% chance of making the school football team. Is this possible? Explain.

4. Mika suggests playing Sasha's game with four slips of paper, numbered 1 to 4. Is Sasha's game now fair? If not, who has the advantage? Explain.

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3.2 Probability and Odds

GOAL

Understand and interpret odds, and relate them to probability.

EXPLORE...

- An oil and vinegar salad dressing is made using 2 parts oil to 1 part vinegar. So, the ratio of oil : vinegar is 2 : 1. What fraction of the dressing is oil?

INVESTIGATE the Math

Suppose that, at the beginning of a regular CFL season, the Saskatchewan Roughriders are given a 25% chance of winning the Grey Cup.

? What are the odds of the Roughriders winning the Grey Cup?

A. What is the event in the situation described above?

B. Express the probability that this event will occur as a fraction out of 100.

C. Describe the **complement** of this event.

D. Express the probability that the complement will occur as a fraction out of 100.

E. Write the **odds in favour** of the Roughriders winning the Grey Cup.

F. Write the **odds against** the Roughriders winning the Grey Cup.

odds in favour
The ratio of the probability that an event will occur to the probability that the event will not occur, or the ratio of the number of favourable outcomes to the number of unfavourable outcomes.

odds against
The ratio of the probability that an event will not occur to the probability that the event will occur, or the ratio of the number of unfavourable outcomes to the number of favourable outcomes.

Reflecting

G. How are the odds in favour of the Roughriders winning related to the odds against them winning?

H. How does the probability of an event and the probability of its complement relate to the odds in favour of the event occurring?

I. How are the odds in favour of an event similar to its probability? How are they different from its probability?

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APPLY the Math

EXAMPLE 1 | Determining odds using sets

Bailey holds all the hearts from a standard deck of 52 playing cards. He asks Morgan to choose a single card without looking.

Determine the odds in favour of Morgan choosing a face card.

Morgan's Solution

Let H be the universal set of all hearts:
$H = \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\}$

I determined and listed the elements in the universal set H.

Let C be the set of face cards that are hearts:
$C = \{J♥, Q♥, K♥\}$

I determined and listed the elements in the subset C. This is the subset that defines the event I was interested in: drawing a face card.

Let C' be the set of cards in the suit of hearts that are not face cards.
$C' = \{A♥, 2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥\}$

Set C is all the face cards that are hearts, which are favourable outcomes. The other cards are in the universal set, but not in set C. They make up the complement of C, or C'. Thus, C' is the set of unfavourable outcomes.

Odds in favour $= n(C) : n(C')$
Odds in favour $= 3:10$

To determine the odds in favour of the event, I counted the number of elements in sets C and C'. Then I formed the ratio of favourable outcomes to unfavourable outcomes.

The odds in favour of choosing a face card are $3:10$.

Your Turn

a) Determine the odds against Morgan drawing a face card.
b) Compare the odds in favour of this event to the odds against it. Do you think Morgan is more likely to draw a face card or something different? Explain.

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EXAMPLE 2 Determining odds from probability

Research shows that the probability of an expectant mother, selected at random, having twins is $\frac{1}{32}$.

a) What are the odds in favour of an expectant mother having twins?
b) What are the odds against an expectant mother having twins?

Lesley's Solution

a) $P(\text{twins}) = \frac{1}{32}$

So, the ratio of twins to all birth combinations is 1 : 32.

I interpreted the probability of having twins. I started by writing this probability as a part-to-whole ratio.

$P(\text{not twins}) = \frac{31}{32}$

So, the ratio of birth combinations that are not twins to all birth combinations is 31 : 32.

I determined the complement of $P(\text{twins})$ and wrote it as a part-to-whole ratio.

The odds in favour of having twins, $P(\text{twins}) : P(\text{not twins})$, are 1 : 31.

I wrote the odds in favour of having twins as a part-to-part ratio. The first term is the number of favourable outcomes from the probability of having twins. The second term is the number of unfavourable outcomes.

b) The odds against having twins, $P(\text{not twins}) : P(\text{twins})$, are 31 : 1.

To determine the odds against having twins, I simply switched the terms in the odds in favour of having twins.

Your Turn

Suppose that the probability of an event happening is $\frac{2}{5}$.

a) What are the odds in favour of the event happening?
b) What are the odds against the event happening?

EXAMPLE 3 Determining probability from odds

A computer randomly selects a university student's name from the university database to award a $100 gift certificate for the bookstore. The odds against the selected student being male are 57 : 43. Determine the probability that the randomly selected university student will be male.

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Cheyenne's Solution

Total number of outcomes $= 57 + 43$
Total number of outcomes $= 100$

I determined a number to represent the total number of possible outcomes by adding the terms for unfavourable and favourable outcomes in the odds against ratio.

For every 100 students, 57 are female and 43 are male.

$$P(\text{male}) = \frac{43}{100}$$

The probability that a randomly selected university student will be male is 43%.

I expressed the probability as a ratio of the number of favourable outcomes to the total number of possible outcomes.

Your Turn

Suppose that the odds in favour of an event are 5 : 3. Is the probability that the event will happen greater or less than 50%?

EXAMPLE 4 Making a decision based on odds and probability

A hockey game has ended in a tie after a 5 min overtime period, so the winner will be decided by a shootout. The coach must decide whether Ellen or Brittany should go first in the shootout. The coach would prefer to use her best scorer first, so she will base her decision on the players' shootout records.

Player	Attempts	Goals Scored
Ellen	13	8
Brittany	17	10

Who should go first?

Coach's Solution

Ellen has 13 attempts and has scored 8 goals. This means that she has $13 - 8$ or 5 attempts where she did not score. The odds in favour of her scoring are 8 : 5.
Brittany has 17 attempts and scored 10 goals, so the odds in favour of her scoring are 10 : 7.

I wrote the odds in favour of each player scoring in a shootout.

Brittany's record looked better, but I was not positive. It was tough to compare the odds in favour, since the ratios involve different numbers.

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The probability that Ellen will score is $\frac{8}{13}$ or about 0.615.

I converted the players' odds in favour of scoring to decimals, so they would be easier to compare.

The probability that Brittany will score is $\frac{10}{17}$ or about 0.588.

Since $0.615 > 0.588$, there is a better chance that Ellen will score. Therefore, Ellen should go first.

Ellen is more likely to score, based on her shootout record.

Your Turn

Josie is also on the team. She has scored twice in three shootout attempts. Should the coach put her first? Explain.

EXAMPLE 5 Interpreting odds against and making a decision

A group of Grade 12 students are holding a charity carnival to support a local animal shelter. The students have created a dice game that they call Bim and a card game that they call Zap. The odds against winning Bim are 5:2, and the odds against winning Zap are 7:3. Which game should Madison play?

Madison's Solution: Changing odds to probability

The odds against winning Bim are 5:2.
The total number of outcomes is $5 + 2$ or 7.
So, if I play Bim 7 times, I am likely to lose 5 times and win 2 times.

I determined the probability that I would win Bim.

$$P(\text{winning Bim}) = \frac{\text{number of wins}}{\text{number of games}}$$

$$P(\text{winning Bim}) = \frac{2}{7}$$

The odds against winning Zap are 7:3.
The total number of outcomes is $7 + 3$ or 10.
So, if I play Zap 10 times, I am likely to lose 7 times and win 3 times.

I used the same method to determine the probability that I would win Zap.

$$P(\text{winning Zap}) = \frac{3}{10}$$

$$P(\text{winning Bim}) = 0.285...$$
$$P(\text{winning Zap}) = 0.3$$

I wrote the fractions as decimals so that I could compare them. It is easier to see that 0.3 is greater than 0.285... than it is to see that $\frac{3}{10}$ is greater than $\frac{2}{7}$.

I should play Zap, since I am more likely to win.

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Samara's Solution: Using equivalent ratios

Odds against winning Bim:

$5:2$

$(5 \times 3):(2 \times 3)$

$15:6$

The odds against winning Bim are $15:6$.

Odds against winning Zap:

$7:3$

$(7 \times 2):(3 \times 2)$

$14:6$

The odds against winning Zap are $14:6$.

I knew that if I wrote the odds ratios as equivalent ratios with the same second term, I would be able to compare the first terms directly.

I multiplied each term in the ratio $5:2$ by 3 and each term in the ratio $7:3$ by 2. The result was two ratios with a second term of 6.

Since $15 > 14$, the odds against winning Bim are greater. Therefore, Madison should play Zap.

The odds against winning Bim are greater than the odds against winning Zap.

Your Turn

The Grade 12 students want to include one more game at their charity carnival. They need to choose between game A and game B. The odds against winning game A are $11:3$, and the odds against winning game B are $17:6$. The goal is to raise as much money as possible for the animal shelter. Which game should the students choose? Assume that people are equally likely to play the two games.

In Summary

Key Ideas

- Odds express a level of confidence about the occurrence of an event.
- The odds in favour of event A occurring are given by the ratio $\frac{P(A)}{P(A')}$ or $P(A):P(A')$.
  This simplifies to the ratio of favourable outcomes to unfavourable outcomes.
- The odds against event A occurring are given by the ratio $\frac{P(A')}{P(A)}$ or $P(A'):P(A)$.
  This simplifies to the ratio of unfavourable outcomes to favourable outcomes.

Need to Know

- $P(A')$ is the probability of the complement of A, where $P(A') = 1 - P(A)$.
- If the odds in favour of event A occurring are $m:n$, then the odds against event A occurring are $n:m$.
- If the odds in favour of event A occurring are $m:n$, then $P(A) = \frac{m}{m+n}$.

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CHECK Your Understanding

1. The odds in favour of Marcia passing her driver's test on the first try are 5 : 3.
- **a)** Determine the odds against Marcia passing her driver's test.
- **b)** Determine the probability that she will pass her driver's test.

2. Colby has 10 coins in his pocket, and 3 of these coins are loonies. He reaches into his pocket and pulls out a coin at random.
- **a)** Determine the probability of the coin being a loonie.
- **b)** Determine the odds against the coin being a loonie.

3. Lily draws a card at random from a standard deck of 52 playing cards.
- **a)** Determine the probability of the card being red.
- **b)** Determine the odds in favour of the card being red.
- **c)** Determine the odds against the card being a spade.
- **d)** Determine the probability of the card being a face card.

PRACTISING

The word "risk" comes from the Latin word for cliff.

4. Mina notices that apple juice is on sale at a local grocery store. The last five times that apple juice was on sale, it was available only twice.
- **a)** Determine the odds in favour of apple juice being available this time.
- **b)** Determine the odds against apple juice being available this time.

5. There are 30 students in Mario's Grade 12 math class. The odds in favour of two students sharing a birthday are 7 : 3. Determine the probability of two students sharing a birthday.

6. Jamia likes to go wall climbing with her friends. In the past, Jamia has climbed to the top of the wall 12 times in 24 attempts.
- **a)** Determine the probability of Jamia climbing to the top this time.
- **b)** Determine the odds against Jamia climbing to the top.
- **c)** The odds from part b) are called "even odds." Explain what this term might mean.

7. The weather forecaster says that there is a 60% probability of snow tomorrow. What are the odds against snow?

8. About 8% of men and 0.5% of women see no difference between the colours red and green. These people are often useful in the military because they can detect khaki camouflage much better than people who do see a difference between red and green. What are the odds in favour of Allan being able to detect camouflage?

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9. Katherine plays ringette. She has scored 4 times in 20 shots on goal. She says that the odds in favour of her scoring are 1 to 5. Is she right? Explain.

10. Jason has been awarded a penalty shot in a hockey game. Gilles is the goalie. Jason has scored 5 times in his last 10 penalty shots. Gilles has blocked 8 of the last 10 penalty shots.

a) Determine the odds in favour of Jason scoring, using his data.

b) Determine the odds in favour of Jason scoring, using Gilles' data.

c) Explain why your answers to parts a) and b) are different.

11. A survey in a Western Canadian city determined that the odds in favour of a person between 18 and 35 using a social networking site are 31 : 19. Determine the probability of a randomly selected person between 18 and 35 using a social networking site.

12. The coach of a basketball team claims that, for the next game, the odds in favour of the team winning are 3 : 2, the odds in favour of the team losing are 1 : 4, and the odds against a tie are 4 : 1. Are these odds possible? Explain.

13. Ratings for the program *Show Trial* indicate that 35% of the viewers are female, 65% are male, 30% are under 18, 20% are 19 to 30 years old, 10% are 30 to 45 years old, and 40% are older than 45. Suppose that someone is watching *Show Trial*.

a) What are the odds in favour of this person being male?

b) What are the odds in favour of this person being older than 45?

14. In a study, 70% of the people who were vaccinated did not get sick, and 42% of the people who were not vaccinated did get sick.

a) What are the odds against getting sick if you are vaccinated?

b) What are the odds against getting sick if you are not vaccinated?

c) Express the odds against from parts a) and b) with the same second term.

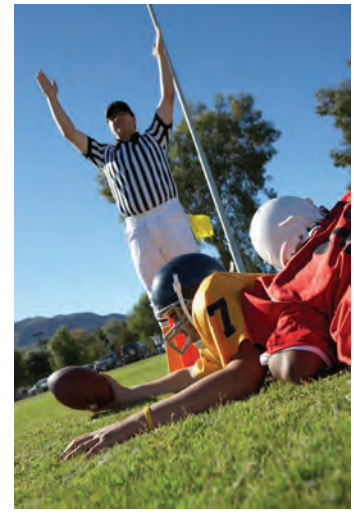
d) Should you be vaccinated? Explain.

15. A high-school football team has the ball at the opponent's 2 yd line. It is the third down. The team is behind by 3 points, with only one second left in the game. The players have two options:

- They can try to score a touchdown. In the past, they have succeeded 5 out of 12 times. If they score a touchdown, they will win the game.
- They can try to kick a field goal. The kicker has scored a field goal from 20 yd or less in 5 of 6 tries. If they score a field goal, they will get 3 points and tie the game, forcing overtime.

a) What are the odds in favour of each option?

b) Which option should the coach choose?



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16. Three people are running for president of the student council. The polls show that Eduard Silvestre has a 45% chance of winning, Julie Jones has a 35% chance of winning, and Bill Black has a 20% chance of winning.

a) What are the odds in favour of each person winning?

b) Suppose that Bill Black withdraws and offers his support to Julie Jones. Further suppose that his supporters also switch to Julie Jones. What are the odds in favour of Julie winning now?

17. Grant is taking a self-study course in fitness training. He must pay $285 to take the final exam. If he fails the exam, he must pay an additional $235 to take it again. The fitness training website lists up-to-date statistics on the pass : fail ratio. The odds that a person with good study habits will pass on his or her first try are 11 : 9. Grant can prepare for the final exam by buying three practice exams for $65.

a) Should Grant buy the practice exams if he has good study habits? Justify your opinion.

b) If the odds in favour of passing on the first try were 17 : 4, should Grant buy the practice exams? What if the odds in favour of passing were 3 : 7? Explain.

18. a) Explain why you can express the odds against an event, A, happening as $P(A') : P(A)$.

b) Suppose that the odds in favour of an event happening are $a : b$. Explain how you can determine the probability of the event happening. Give an example.

c) Suppose that the probability of an event happening is $\frac{a}{c}$. Explain how you can determine the odds against the event happening. Give an example.

Closing

19. Do you prefer to express the likelihood that an event will happen using probability or odds? Explain why, and provide an example.

Extending

20. The probability that a child between the ages of 6 and 18 will need corrective lenses to see properly is 25.4%. Of the children between the ages of 6 and 18 who do need corrective lenses, the odds in favour of them being girls are 141 : 100.

a) Determine the probability that a randomly selected girl between the ages of 6 and 18 will need corrective lenses.

b) Determine the odds that a randomly selected 18-year-old boy will need corrective lenses.

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3.3 Probabilities Using Counting Methods

GOAL

Solve probability problems that involve counting techniques.

YOU WILL NEED

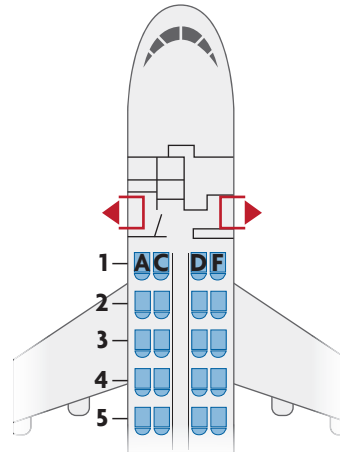
- calculator

INVESTIGATE the Math

As a volunteer activity, 10 students want to put on a talent show at a retirement home. To organize the show, 3 of these students will be chosen at random to form a committee. Victoria really wants to be on this committee, since her grandmother lives at the home. Each student's name will be written on a slip of paper and placed in a hat. Then 3 names will be drawn.

EXPLORE...

- Ari, Ben, Cam, and Dan have booked a flight to Florida. The airplane has four seats in each row, and they are planning to sit in the same row. What is the probability that Cam and Dan will be sitting on the same side of the airplane?



? What is the likelihood that Victoria's name will be drawn from the hat?

A. Is Victoria's name just as likely to be drawn as any other name? Explain.

B. Does the order in which the names are drawn matter? Explain.

C. In how many different ways can 3 names be drawn from a hat with 10 names? Explain.

D. In how many different ways can Victoria's name be drawn with 2 other names? Explain.

E. What is the probability that Victoria's name will be drawn? Explain.

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Reflecting

F. Did you assume that this problem involved a situation with or without replacement? Explain.

G. Suppose that only 8 students, including Victoria, volunteered for the talent show. Would her name be more likely or less likely to be drawn from the hat? Justify your decision.

H. Suppose that only 2 of the 10 names will be drawn from the hat, instead of 3 names. Is Victoria's name more likely or less likely to be drawn? Justify your decision.

APPLY the Math

EXAMPLE 1 Solving a probability problem using counting techniques

Jamaal, Ethan, and Alberto are competing with seven other boys to be on their school's cross-country team. All the boys have an equal chance of winning the trial race. Determine the probability that Jamaal, Ethan, and Alberto will place first, second, and third, in any order.

Brandi's Solution: Using permutations

Jamaal, Ethan, and Alberto can place first, second, or third, in any order. There are ${}_3P_3$ ways in which three runners can place in three positions.

I determined the number of ways in which Jamaal, Ethan, and Alberto can finish in the top three positions.

$${}_3P_3 = \frac{3!}{(3-3)!}$$

$${}_3P_3 = \frac{3 \cdot 2 \cdot 1}{1}$$

$${}_3P_3 = 6$$

There are 6 favourable outcomes.

There are ${}_{10}P_3$ ways that 10 runners can place first, second, or third.

I determined the total number of ways that 10 runners can place first, second, and third.

$${}_{10}P_3 = \frac{10!}{(10-3)!}$$

$${}_{10}P_3 = \frac{10!}{7!}$$

$${}_{10}P_3 = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$$

$${}_{10}P_3 = 10 \cdot 9 \cdot 8$$

$${}_{10}P_3 = 720$$

There are 720 possible outcomes.

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$$P(\text{J, E, and A place 1, 2, or 3}) = \frac{{}_3P_3}{{}_{10}P_3}$$

I divided the number of favourable outcomes by the total number of outcomes.

$$P(\text{J, E, and A place 1, 2, or 3}) = \frac{6}{720} \text{ or } \frac{1}{120}$$

The probability that Jamaal, Ethan, and Alberto will place in the top three positions is $\frac{1}{120}$ or about 0.83%.

Davinder's Solution: Using combinations

Jamaal, Ethan, and Alberto can place first, second, or third, in any order.
There are ${}_3C_3$ ways in which only these three runners place in the top three positions.

I determined the number of combinations in which Jamaal, Ethan, and Alberto finish in the top three positions.

$${}_3C_3 = \frac{3!}{3!(3-3)!}$$

$${}_3C_3 = \frac{3!}{3! \cdot 0!}$$

$${}_3C_3 = \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 1}$$

$${}_3C_3 = 1$$

There is only 1 favourable outcome.

$${}_{10}C_3 = \frac{10!}{3!(10-3)!}$$

I determined the total number of combinations that are possible for all 10 runners placing in first, second, and third.

$${}_{10}C_3 = \frac{10!}{3! \cdot 7!}$$

$${}_{10}C_3 = \frac{10 \cdot 9 \cdot 8}{3!}$$

I simplified by dividing both the numerator and denominator by 7!, then I divided 9 by 3 and 8 by 2.

$${}_{10}C_3 = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$$

$${}_{10}C_3 = 10 \cdot 3 \cdot 4$$

$${}_{10}C_3 = 120$$

There are 120 possible outcomes.

$$P(\text{J, E, and A place 1st, 2nd, or 3rd}) = \frac{{}_3C_3}{{}_{10}C_3}$$

To determine probability, I divided the number of favourable outcomes by the total number of outcomes.

$$P(\text{J, E, and A place 1st, 2nd, or 3rd}) = \frac{1}{120}$$

The probability that Jamaal, Ethan, and Alberto will place in the top three positions is $\frac{1}{120}$ or about 0.83%.

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Your Turn

Suppose that Zachary is also trying out for the team, so now there will be 11 runners in the trial race. What is the probability that three of Jamal, Ethan, Alberto, and Zachary will place in the top three positions?

EXAMPLE 2 Solving a probability problem with the Fundamental Counting Principle

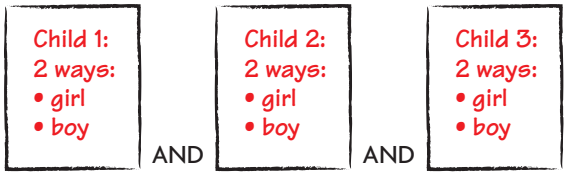
About 20 years after they graduated from high school, Blake, Mario, and Simon met in a mall. Blake had two daughters with him, and he said he had three other children at home. Determine the probability that at least one of Blake's children is a boy.

Mario's Solution: Using indirect reasoning

I considered the genders of the children at home.

I knew that the genders of the children at the mall had nothing to do with the genders of the children at home. I just needed to calculate the probability that the three children at home are girls or boys.

Each child is either a boy or a girl.



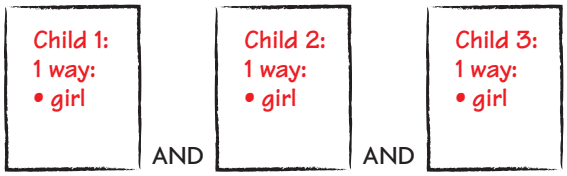
Let C represent the total possible outcomes.

$C = 2 \cdot 2 \cdot 2$

$C = 8$

There are 8 possible outcomes.

I determined the total number of possible outcomes using the **Fundamental Counting Principle**.



Let G represent the number of outcomes where each child is a girl.

$G = 1 \cdot 1 \cdot 1$

$G = 1$

In only 1 outcome is each child a girl.

My result made sense because there is only one way that all three children can be girls.

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$$P(\text{all girls}) = \frac{n(\text{children at home all girls})}{n(\text{all possibilities})}$$

I wrote the probability.

$$P(\text{all girls}) = \frac{1}{8}$$

The probability that all five children are girls is $\frac{1}{8}$ or 12.5%.

$$P(\text{at least one boy}) = 1 - P(\text{all girls})$$

The probability of at least one boy is the complement of the probability of all girls.

$$P(\text{at least one boy}) = 1 - \frac{1}{8}$$

$$P(\text{at least one boy}) = \frac{7}{8}$$

The probability that at least one child is a boy is $\frac{7}{8}$ or 87.5%.

Simon's Solution: Using direct reasoning

I let U represent the total number of possible outcomes:

$U = 2 \cdot 2 \cdot 2$

$U = 8$

There are 8 possible outcomes for the genders of the children at home.

I knew that the children at the mall were girls, so I needed to consider only the genders of the children at home.

Each child at home is either a girl or a boy.

There are three cases in which at least one child is a boy. I wrote out the possible ways for each case.

In these cases, each child is unique, which means that order is important, so the problem involves permutations.

Case 1: exactly one boy
- boy, girl, girl
- girl, boy, girl
- girl, girl, boy

There can be one boy in 3 ways.

If there is one boy, he must be the eldest, middle, or youngest of the three children at home.

Case 2: exactly two boys
- girl, boy, boy
- boy, girl, boy
- boy, boy, girl

There can be two boys in 3 ways.

If there are two boys, then the girl must be the eldest, middle, or youngest of the three children at home.

Case 3: exactly three boys
- boy, boy, boy

There can be three boys in 1 way.

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I let B represent the number of ways in which at least one child is a boy:

$B = 3 + 3 + 1$

$B = 7$

There are 7 ways in which at least one child is a boy.

I determined the total number of ways in which at least one child is a boy.

$$P(\text{at least one boy}) = \frac{B}{U}$$

$$P(\text{at least one boy}) = \frac{7}{8}$$

I wrote the probability.

The probability that at least one child is a boy is $\frac{7}{8}$ or 87.5%.

Your Turn

Suppose that Blake had had one daughter with him at the mall and four children at home. Determine the probability of each event.

a) All five of Blake's children are girls.

b) At least one of Blake's children is a boy.

EXAMPLE 3 Solving a probability problem using reasoning

Beau hosts a morning radio show in Saskatoon. To advertise his show, he is holding a contest at a local mall. He spells out SASKATCHEWAN with letter tiles. Then he turns the tiles face down and mixes them up. He asks Sally to arrange the tiles in a row and turn them face up. If the row of tiles spells SASKATCHEWAN, Sally will win a new car. Determine the probability that Sally will win the car.

Sally's Solution

S A S K A T C H E W A N

There are 12 letters in total: 2 Ss, 3 As, and 7 other letters.

I examined the letters in the word.

Since the order must be correct, this problem involves permutations.

I let L represent the total number of ways in which I can arrange the letters:

$$L = \frac{12!}{2! \cdot 3!}$$

$L = 39\,916\,800$

This is the total number of outcomes.

I could arrange the 12 letters in 12! ways if they were all different. But there are 2 Ss and 3 As. I divided by 2! and by 3! to eliminate the arrangements that would be the same.

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I let R represent the number of ways that I can spell SASKATCHEWAN.

$R = 1$

I can spell SASKATCHEWAN in just 1 way.

So, there is only 1 favourable outcome.

Since I considered only the unique arrangements of the 12 letters to determine the total number of outcomes, there is only 1 correct way to spell SASKATCHEWAN.

$$P(\text{winning the car}) = \frac{R}{L}$$

I determined the probability that I will win.

$$P(\text{winning the car}) = \frac{1}{39\,916\,800}$$

The probability that I will win the car is $\frac{1}{39\,916\,800}$

Winning the car is very unlikely.

Your Turn

Suppose that the contest word was SASKATOON. Would Sally's probability of winning be greater or less? Explain.

EXAMPLE 4 Solving a probability problem with conditions

There are 18 bikes in Marnie's spinning class. The bikes are arranged in 3 rows, with 6 bikes in each row. Allison, Brett, Carol, Doug, Erica, and Franco each call the gym to reserve a bike. They hope to be in the same row, but they cannot request a specific bike. Determine the probability that all 6 friends will be in the same row, with Allison and Franco at either end.

Marnie's Solution

There is an equal likelihood that any of the 18 bikes will be assigned to a participant, since specific bikes cannot be requested.

I assumed that the bikes are assigned randomly. I also assumed that all 18 bikes will be used.

Let F represent the event that the 6 friends are seated in any row:

2	4	3	2	1	1
12	11	10	9	8	7
6	5	4	3	2	1

I determined the number of ways in which the 6 friends can be assigned bikes in the first row, if Allison and Franco are at either end.

Since order is important, this problem involves permutations.

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- The number of ways to seat Allison and Franco at either end is 2! or ${}_2P_2$.
- The number of ways to seat the other 4 friends is 4! or ${}_4P_4$.
- The number of ways to seat the other 12 people in the class is 12! or ${}_{12}P_{12}$.

The total number of ways to seat the 6 friends in the first row is $({}_2P_2)({}_4P_4)({}_{12}P_{12})$.

I knew that I could multiply here, since I was seating Allison and Frank on the ends AND the other 4 friends in between AND the remaining 12 people.

The total number of ways to seat the 6 friends in any row is $3({}_2P_2)({}_4P_4)({}_{12}P_{12})$.

Since the 6 friends can be seated in the same row 3 different ways (row 1, 2, or 3), I multiplied the number of ways they can sit in the first row by 3.

The total number of ways to assign 18 people to 18 bikes is 18! or ${}_{18}P_{18}$.

The number of ways that the 18 bikes can be assigned is equivalent to the number of possible permutations for a set of 18 objects.

$$P(F) = \frac{3({}_2P_2)({}_4P_4)({}_{12}P_{12})}{{}_{18}P_{18}}$$

I determined the probability that the 6 friends will be in the same row by dividing the number of favourable outcomes by the total number of outcomes.

$$P(F) = \frac{3 \cdot 2! \cdot 4! \cdot 12!}{18!}$$

$$P(F) = \frac{3 \cdot 2! \cdot 4! \cdot 12!}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}$$

I used the fact that $\frac{12!}{12!} = 1$ to simplify.

$$P(F) = \frac{3 \cdot 2! \cdot 4!}{18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13}$$

$$P(F) = \frac{144}{13\,366\,080}$$

I used my calculator to multiply since the numbers were large.

$$P(F) = \frac{1}{92\,820}$$

The probability that all 6 friends will be in the same row, with Allison and Franco at either end, is $\frac{1}{92\,820}$.

Your Turn

Franco determined the solution to this problem by calculating $\frac{3 \cdot 2 \cdot {}_4P_4}{{}_{18}P_6}$.

Would this calculation give the correct answer? Explain.

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In Summary

Key Idea

- You may be able to use the Fundamental Counting Principle and techniques involving permutations and combinations to solve probability problems with many possible outcomes. The context of each particular problem will determine which counting techniques you will use.

Need to Know

- Use permutations when order is important in the outcomes.
- Use combinations when order is not important in the outcomes.

CHECK Your Understanding

1. A credit card company randomly generates temporary four-digit pass codes for cardholders. Suri is expecting her credit card to arrive in the mail. Determine the probability that her pass code will consist of four different even digits.

2. In a card game called Crazy Eights, players are dealt 8 cards from a standard deck of 52 playing cards. Determine the probability that a hand will consist of 8 hearts.

3. From a committee of 12 people, 2 of these people are randomly chosen to be president and secretary. Determine the probability that Ben and Jen will be chosen.

PRACTISING

4. Five boys and six girls have signed up for a trip to see Francophone artists compete at Festival International de la Chanson de Granby. Only four students will be selected to go on the trip. Determine the probability for the following:

a) Only boys will be on the trip.

b) There will equal numbers of boys and girls on the trip.

c) There will be more girls than boys on the trip.

5. Access to a particular online game is password protected. Every player must create a password that consists of two capital letters followed by three digits. For each condition below, determine the probability that a password chosen at random will contain the letters S and Q.

a) Repetitions are not allowed in a password.

b) Repetitions are allowed in a password.

Festival International de la Chanson de Granby is the leading French language song competition in the country. Previous winners have included Senaya, of Montréal, and Steeve Thomas, of Vancouver.

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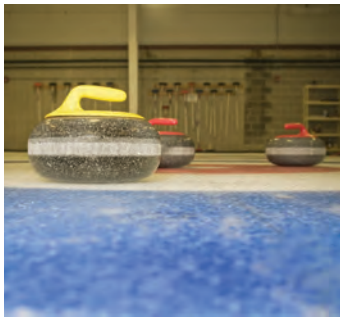

6. A high-school athletics department is forming a beginners curling team to play in a social tournament. Nine students, including you and your three friends, have signed up for the four positions of skip, third, second, and lead. The positions will be filled randomly, so every student has an equal chance of being chosen for any position.

a) Determine the probability that you and your three friends will be chosen.

b) How would this probability change if only eight students had signed up for the team?

7. There are nine players on a baseball team, all with roughly equal athletic ability. The coach has decided to choose the players who will play the four infield positions (first base, second base, third base, and shortstop) randomly. Tara and Laura are on the team. Determine the odds in favour of Tara and Laura being chosen to play in the infield.

8. A student council has 15 members, including Yuko, Luigi, and Justin.

a) The staff advisor will select three members at random to be treasurer, secretary, and liaison to the principal. Determine the probability that the staff advisor will select Yuko to be treasurer, Luigi to be secretary, and Justin to be liaison.

b) The staff advisor will also select three members at random to clean up after the pep rally. Determine the probability that the staff advisor will select Yuko, Luigi, and Justin to do this.

9. Lesley needs to create a four-digit password to access her voice mail. She can repeat some of the digits, but all four digits cannot be the same.

a) Determine the probability that her password will be greater than 5000.

b) Determine the probability that the first and last digits of her password will be 4.

c) Determine the probability that the first digit of her password will be odd and the last digit will be even.

10. A student council consists of 16 girls and 7 boys. To form a subcommittee, 5 students are randomly selected from the council. Determine the odds in favour of 3 girls and 2 boys being on the subcommittee.

11. Larysa tosses four coins. Determine the probability that at least one coin will land as tails.

12. Five friends, including Bilyana and Bojana, are sitting in a row in a theatre.

a) Determine the probability that Bilyana and Bojana are sitting together.

b) Determine the probability that they are not sitting together.

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13. Tanya is planning her schedule for university. She wants to take the following courses during her first two terms: biology, English, psychology, religion, linear algebra, political studies, economics, and philosophy. She is equally likely to take any of these courses in either term, since they are all introductory courses.

a) Suppose that Tanya decides to take four courses in her first term. Determine the probability that three of them will be psychology, linear algebra, and English.

b) Suppose that Tanya decides to take five courses in her first term. Determine the probability that three of them will be religion, political studies, and biology.

14. Doc deals you eight cards at random from a standard deck of 52 playing cards. Determine the probability that you have the following hands.

a) A, 2, 3, 4, 5, 6, 7, 8 of the same suit

b) Eight cards of the same colour

c) Four face cards and four other cards

15. Erynn has letter tiles that spell CABINET. She has selected three of these tiles at random. Determine the probability that the tiles she selected are two vowels and one consonant.

16. At a local dog show, dogs compete in eight different categories. The eight winners of these categories are all different breeds, including a sheltie and a bearded collie. The organizers randomly line up the eight winners for the "Best in Show" competition. Determine the probability that the bearded collie and the sheltie will be next to each other in the lineup.

17. The starting lineup for a basketball team consists of two guards and three forwards. On the team that sisters Maggie and Tanya play for, there are seven forwards and five guards from which the coach can choose a starting lineup. Maggie is a guard and Tanya is a forward. For the first exhibition game of the year, the coach will select the starting players at random. What is the probability that both Maggie and Tanya will be in the starting lineup?

Closing

18. Explain when you would use permutations to solve a probability problem and when you would use combinations. Give an example.

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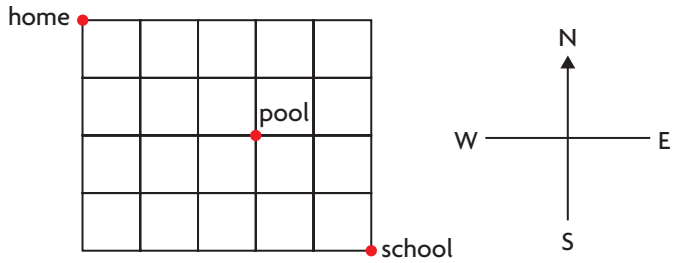
Extending

19. Tuyet likes to vary her walk to school. At each intersection, she randomly walks either south or east. Determine the probability that she will pass the pool on her way to school.



20. In Chapter 2, page 95, you learned about birthday permutations. Determine how many people would need to be in a room for the probability of two people having the same birthday to be 80%.

History | Connection

Counter Intuition

Which game would you rather play?

Game 1: Roll a single die 4 times. If a 6 comes up, you win a point. If not, you lose a point.
Game 2: Roll two dice 24 times. If a double 6 comes up, you win a point. If not, you lose a point.

Around 1650, the Chevalier de Méré, Antoine Gombaud, said that these two games gave you the same chance of winning. His reasoning was as follows:

- With game 1, the chance of rolling a 6 in 4 rolls is $4\left(\frac{1}{6}\right)$ or $\frac{2}{3}$.
- With game 2, the chance of rolling two 6s in 24 rolls is $24\left(\frac{1}{36}\right)$ or $\frac{2}{3}$.

But when the Chevalier de Méré played game 2, he lost quite often. He asked the mathematician Blaise Pascal to explain why. After much analysis, Pascal realized that the probability of winning game 2 is actually less than 50%. Pascal's analysis of this situation laid the groundwork for the study of probability.

A. Determine why you have a better chance of winning game 1 than game 2.

Blaise Pascal (1623–1662) was a brilliant mathematician, physicist, and inventor. At 18, he invented a calculating device. He is also said to have been the first to wear a wrist watch.

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3 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: How are probability and odds similar and different?

A: Probability compares the likelihood that an event will happen to all possible outcomes, including the event itself. The odds in favour of an event happening compare the likelihood that an event will happen to the likelihood that it will not happen. The odds against an event happening compare the likelihood that an event will not happen to the likelihood that it will happen. For example, suppose that Harry is late for the bus 7 out of 10 school days. The probability that he will be late is $\frac{7}{10}$. If he is late 7 times, then he is not late 3 times. The odds in favour of Harry being late are $7:3$. The odds against Harry being late are $3:7$.

Study Aid

- See Lesson 3.2, Examples 2 and 3.
- Try Mid-Chapter Review Questions 1 and 2.

Q: How can you determine the probability and odds of an event, C?

A: You can use a Venn diagram to create and study subset C. Indicate the number of elements in subset C as a. Indicate the number of elements in subset C' as b. The probability is the number of elements in the subset divided by the number of elements in the universal set, or

$$P(C) = \frac{a}{a + b}$$

The odds in favour is the ratio of the number of favourable outcomes, a, to the number of unfavourable outcomes, b. You can write this in one of two ways:

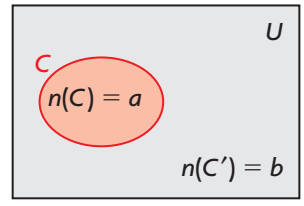
$$a:b \text{ or } \frac{a}{b}$$

The odds against is the ratio of number of unfavourable outcomes to the number of favourable outcomes. For the odds against, write the reciprocal of the odds in favour:

$$b:a \text{ or } \frac{b}{a}$$

Study Aid

- See Lesson 3.2, Examples 1 to 4.
- Try Mid-Chapter Review Question 3.



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For example, suppose that Brent draws a card at random from a suit of 13 hearts. Let E represent the set of cards with even numbers:

$$E = \{2\heartsuit, 4\heartsuit, 6\heartsuit, 8\heartsuit, 10\heartsuit\}$$
$$E' = \{A\heartsuit, 3\heartsuit, 5\heartsuit, 7\heartsuit, 9\heartsuit, J\heartsuit, Q\heartsuit, K\heartsuit\}$$

$$n(E) = 5, n(E') = 8$$

The probability that Brent will draw a card with an even number is:

$$P(E) = \frac{5}{(5 + 8)} \text{ or } \frac{5}{13}$$

The odds in favour of drawing a card with an even number are 5 : 8. The odds against drawing a card with an even number are 8 : 5.

Study **Aid**

- See Lesson 3.3, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 6 to 9.

Q: How are counting techniques used to solve probability problems?

A: If order is important, use a strategy that involves permutations. If order is not important, use a strategy that involves combinations. Determine the probability of the event by dividing the number of ways that the favourable outcome can occur by the total number of possible outcomes.

For example, suppose that 13 students volunteer to help at a car wash but only 9 will be chosen. Determine the probability that Anne and David will be chosen. This is a combination because the order in which the students are chosen is not important. Therefore, the total number of ways that the 9 students can be chosen is ${}_{13}C_9$. There is only one way that Anne and David can be chosen. There are 7 other students who will be chosen from the remaining 11 volunteers, so the number of ways that Anne and David can be chosen is ${}_2C_2 \cdot {}_{11}C_7$.

$$P(\text{Anne and David chosen}) = \frac{{}_2C_2 \cdot {}_{11}C_7}{{}_{13}C_9}$$

$$P(\text{Anne and David chosen}) = \frac{330}{715}$$

The probability that Anne and David will be chosen among the 9 volunteers selected is $\frac{300}{715}$.

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PRACTISING

Lesson 3.1

1. Decide whether each game is fair or not fair. If it is not fair, who has the advantage? Explain.
- **a)** Erik and Ethan roll three four-sided dice. If the sum is greater than the product, Erik wins a point. If the sum is not greater than the product, Ethan wins a point.
- **b)** Bob and Anne each toss a dime and a nickel. If both coins land as heads, Bob wins a point. If both coins land as tails, Anne wins a point. Otherwise, no one wins a point.

2. Garnet plays a board game that involves two spinners: one with the numbers 1 to 5, and the other with the numbers 1 to 4. The sum of the two numbers determines the player's next move.
- **a)** Create a sample space for the spinners.
- **b)** Determine the probability that each of the following events will happen:
  - **i)** If the sum is 5, the player loses a turn.
  - **ii)** If the sum is 8 or 9, the player moves 10 squares.
  - **iii)** If the sum is 4, the player moves back 5 squares.

Lesson 3.2

3. Fatouma draws a card at random from a standard deck of 52 playing cards.
- **a)** Determine the probability of the card being black.
- **b)** Determine the odds in favour of the card being black.
- **c)** Determine the odds against the card being a diamond.
- **d)** Determine the probability of the card not being a face card.

4. A junior golfer enters a tournament. According to his recent performance against the other players, the odds against him winning are 23 : 5. Determine the probability that he will win the tournament.

5. In a group of 22 people chosen at random, the probability of two people having the same birthday is slightly less than 50%. As of 2010, Canada has had 22 prime ministers.
- **a)** Determine the odds in favour of two of our prime ministers having the same birthday.
- **b)** Determine the odds against two of our prime ministers having the same birthday.
- **c)** Investigate to see whether two of our prime ministers did have the same birthday.

Lesson 3.3

6. Three people are randomly chosen from a fellowship of 14 people to be president, treasurer, and secretary. What is the probability that Frodo, Sam, and Aragorn will be the ones chosen?

7. Euchre is played with a deck of 24 cards that is similar to a standard deck of 52 playing cards, but with only the ace (A), 9, 10, jack (J), queen (Q), king (K) for all four suits. Each player is dealt five cards. Determine the probability that a dealt hand will contain the following:
- **a)** A, K, Q, J of the same suit
- **b)** Five cards of the same colour
- **c)** Four of a kind (that is, four cards of the same rank)

8. Emanuella has a music player that shuffles the songs so that they are played in a random order. Of the 30 songs in a playlist, 6 are her favourites. What is the probability that these 6 songs will all be played together the next time she listens to the playlist? Assume that she will listen to all 30 songs.

9. Stella has 12 coins in her hands, and 6 of these coins are loonies. She accidentally (at random) drops 3 of the coins. Determine the probability that exactly 2 of the dropped coins are loonies.

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3.4 Mutually Exclusive Events

YOU WILL NEED

- calculator

GOAL

Understand and solve problems that involve mutually exclusive and non-mutually exclusive events.

INVESTIGATE the Math

EXPLORE...

- Carlos drew a single card from a standard deck of 52 playing cards. What is the probability that the card he drew is either an 8 or a black card?

Janek and Violeta are playing a board game. To move on this turn, Janek must roll either doubles or a sum of 7 with the two dice.

? What is the probability that Janek will move on this turn?

A. Create an outcome table for the sample space of a pair of dice.

B. Circle each occurrence of throwing doubles in your outcome table. Determine P(throw doubles).

C. Shade each occurrence of rolling a sum of 7 in your table. Determine P(sum of 7).

D. Sum the probabilities of the two events in parts B and C.

E. Use your outcome table to determine the probability that Janek will roll doubles or roll a sum of 7. How does this probability relate to the sum of the probabilities in part D?

F. Illustrate the sample space in a Venn diagram.

G. Are the two events **mutually exclusive**? How does your Venn diagram show this? How does your outcome table show this?

H. Determine P(throw doubles $\cup$ sum of 7) using your Venn diagram.

Reflecting

I. Given that A and B are mutually exclusive events, write a formula you could use to determine $P(A \cup B)$ in terms of $P(A)$ and $P(B)$.

J. Suppose that Janek has to roll either doubles (event A) or a sum of 6 (event B) to move. Could you use the formula you wrote in part I to determine if Janek can move? If not, what formula could you use? Explain.

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APPLY the Math

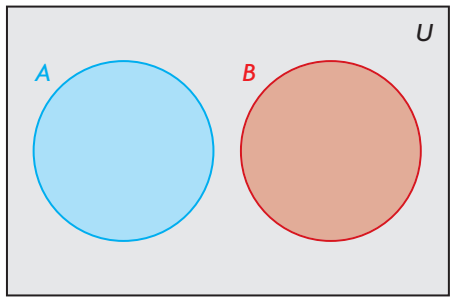
EXAMPLE 1 Reasoning to develop a formula for events that are mutually exclusive

Jamie claims that

$$P(A \cup B) = P(A) + P(B)$$

where A and B represent mutually exclusive events. Is she correct? Explain.

Cyndie's Solution



A and B represent mutually exclusive events, so their sets of favourable outcomes are disjoint. That is, they have no common elements. I drew a Venn diagram to show this.

$$P(A \cup B) = \frac{n(A \cup B)}{n(U)}$$

Since A and B are mutually exclusive, there is no intersection. I did not need to use the **Principle of Inclusion and Exclusion.**

$$P(A \cup B) = \frac{n(A) + n(B)}{n(U)}$$

Since A and B are mutually exclusive, there is no intersection to subtract.

The number of elements in the union of A and B is equal to the number of elements in A plus the number of elements in B.

$$P(A \cup B) = \frac{n(A)}{n(U)} + \frac{n(B)}{n(U)}$$

Since the numerator is the sum of two terms, I can write the fraction as the sum of two fractions with the same denominator.

$$P(A \cup B) = P(A) + P(B)$$

The probability of A is equal to the number of elements in A divided by the number of elements in U.

Similarly, the probability of B is equal to the number of elements in B divided by the number of elements in U.

Jamie is correct and is supported by my reasoning shown above.

Your Turn

Do you think this formula is true for two events that are not mutually exclusive? Explain.

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EXAMPLE 2 Determining the probability of events that are not mutually exclusive

Recall the board game that Janek and Violeta were playing. According to a different rule, if a player rolls a sum that is greater than 8 or a multiple of 5, the player gets a bonus of 100 points.

a) Determine the probability that Violeta will receive a bonus of 100 points on her next roll.

b) Write a formula you could use to calculate the probability of two non-mutually exclusive events. Answer part a) again to show that your formula works.

Violeta's Solution

a)

Possible Sums When a Pair of Dice are Rolled						
Die 1/ Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

I created an outcome table for the sample space of the possible sums.

I wrote the sums greater than 8 in red, and I highlighted the multiples of 5.

Three numbers were both red and highlighted. Therefore, the events are not mutually exclusive.

Let A represent rolling a sum greater than 8.
Let B represent rolling a sum that is a multiple of 5.

I used A and B to represent the two events.

$$P(A) = \frac{10}{36}$$

$$P(B) = \frac{7}{36}$$

I used my outcome table to determine the probability of each event. I did not write $P(A)$ in least terms, because it is easier to work with fractions that have the same denominator.

Since 14 numbers are red or highlighted, or both,

$$P(A \cup B) = \frac{14}{36}$$

$$P(A \cup B) = \frac{7}{18}, \text{ or } 0.388...$$

I determined the probability of A or B using my sample space.

There is about a 39% chance that Violeta will receive the bonus on her next roll.

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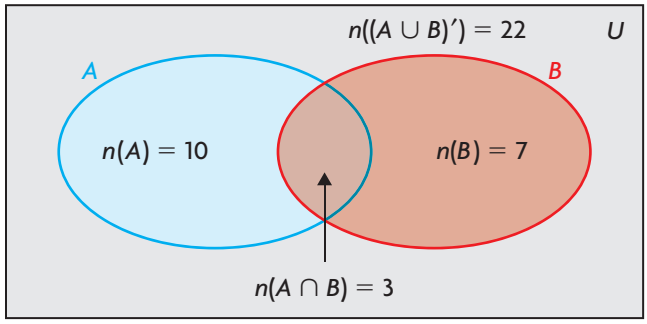
b) Let U represent the universal set of all possible sums when a pair of dice are rolled.



I drew a Venn diagram to visualize the problem. I wrote the number of elements for each event in the appropriate place. There are three sums of 10 that belong to both event A and event B.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

I knew that the number of elements in the union of two non-disjoint sets is the number of elements in the intersection of the two sets subtracted from the sum of the number of elements in each set.

$$P(A \cup B) = \frac{n(A) + n(B) - n(A \cap B)}{n(U)}$$

$$P(A \cup B) = \frac{n(A)}{n(U)} + \frac{n(B)}{n(U)} - \frac{n(A \cap B)}{n(U)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

I wrote a formula for the probability of two non-mutually exclusive events.

I used my Venn diagram to determine the number of favourable outcomes.

The total number of outcomes is the number of elements in the universal set.

$$P(A \cup B) = \frac{10}{36} + \frac{7}{36} - \frac{3}{36}$$

$$P(A \cup B) = \frac{14}{36}$$

$$P(A \cup B) = \frac{7}{18}, \text{ or } 0.388...$$

I determined the probability using my formula. I got the same result from the outcome table, which supports the formula.

There is about a 39% chance that Violeta will receive the bonus on her next roll.

Your Turn

a) Would Violeta's formula also work for mutually exclusive events? Explain.

b) Bronwyn determined $P(A \cup B)$ using the following formula:

$$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

Would this formula give the correct answer? Explain.

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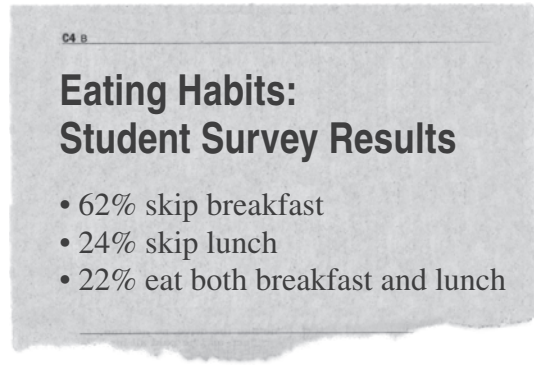
EXAMPLE 3 Using a Venn diagram to solve a probability problem that involves two events

A school newspaper published the results of a recent survey.

a) Are skipping breakfast and skipping lunch mutually exclusive events?

b) Determine the probability that a randomly selected student skips breakfast but not lunch.

c) Determine the probability that a randomly selected student skips at least one of breakfast or lunch.


C4 B

Eating Habits: Student Survey Results

- 62% skip breakfast
- 24% skip lunch
- 22% eat both breakfast and lunch

Mei Lin's Solution

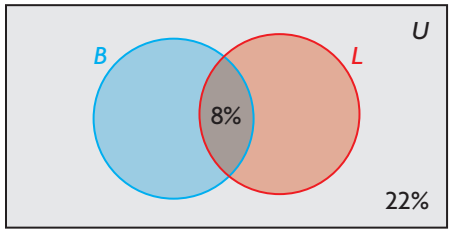
a) If the events of skipping breakfast and skipping lunch are mutually exclusive, then the sum of the probabilities of these events and the sum of the probability of their complement (eat both breakfast and lunch) should total 100%. But, 62% + 24% + 22% is 108%.
This means that 8% of the students surveyed were counted twice. The events are not mutually exclusive.

Since the only two options are eating a meal or skipping a meal, the results account for everyone who was surveyed.

My analysis of the survey data shows that some of the students surveyed must skip both breakfast and lunch.

b) Let B represent skipping breakfast.
Let L represent skipping lunch.

I used B and L to represent the two events.



I drew a Venn diagram to show the survey results.

I knew that the probability of skipping breakfast and lunch is 8%.

$$P(B \setminus L) = P(B) - P(B \cap L)$$
$$P(B \setminus L) = 62\% - 8\%$$
$$P(B \setminus L) = 54\%$$

The probability of skipping only breakfast is 54%.

I knew that I could determine the probability of skipping breakfast but not lunch by subtracting the probability of skipping both meals from the probability of skipping breakfast.

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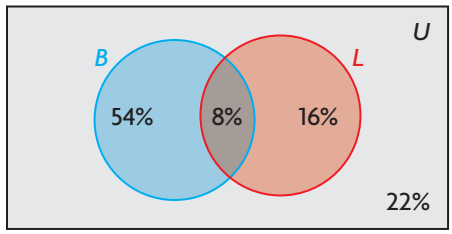
c) $P(L \setminus B) = P(L) - P(B \cap L)$
$P(L \setminus B) = 24\% - 8\%$
$P(L \setminus B) = 16\%$

I used the same reasoning to determine the probability of skipping lunch but not breakfast.



I updated my Venn diagram.

$P(B \cup L) = P(B \setminus L) + P(L \setminus B) + P(B \cap L)$
$P(B \cup L) = 54\% + 16\% + 8\%$
$P(B \cup L) = 78\%$

According to this survey, there is a 78% chance that a student selected at random will skip breakfast or lunch.

I determined the probability of skipping at least one of breakfast or lunch by adding the three areas of my Venn diagram.

"Skips at least one of breakfast or lunch" means the same as skips one or the other, or both. Since 22% eat both breakfast and lunch, skips one or the other, or both is just 100% − 22% or 78%. This supports my solution.

Your Turn

Suppose that the school cafeteria introduces a different lunch menu. Now only 14% of the students skip lunch, but 62% of the students continue to skip breakfast. Determine the maximum percent and minimum percent of students who now eat both breakfast and lunch. Provide your reasoning.

EXAMPLE 4 Making a decision based on probability

Reid's mother buys a new washer and dryer set for $2500 with a 1-year warranty. She can buy a 3-year extended warranty for $450. Reid researches the repair statistics for this washer and dryer set and finds the data in the table below. Should Reid's mother buy the extended warranty? Justify your decision.

Appliance	*P*(repair within extended warranty period)	Average Repair Cost
washer	22%	$400
dryer	13%	$300
both	3%	$700

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Reid's Solution

$W = \{\text{washer needs repair}\}$
$D = \{\text{dryer needs repair}\}$
$P(W \cap D) = 3\%$

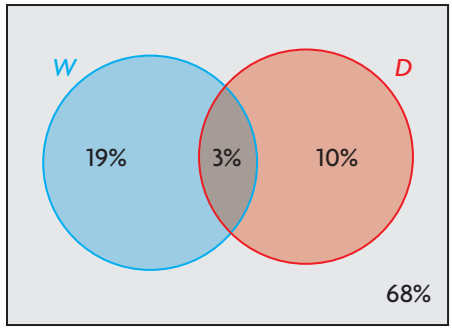
The probability that both appliances will need repair is 3%.

I determined the probability that only the washer will need repair.

$P(W \setminus D) = P(W) - P(W \cap D)$
$P(W \setminus D) = 22\% - 3\%$
$P(W \setminus D) = 19\%$

I determined the probability that only the dryer will need repair.

$P(D \setminus W) = P(D) - P(W \cap D)$
$P(D \setminus W) = 13\% - 3\%$
$P(D \setminus W) = 10\%$



I drew a Venn diagram to visualize the probability of needing repair.

$P(W \cup D) = P(W \setminus D) + P(D \setminus W) + P(W \cap D)$
$P(W \cup D) = 19\% + 10\% + 3\%$
$P(W \cup D) = 32\%$

I determined the probability that at least one appliance will need repair by adding the individual probabilities.

There is a 32% chance repair will be needed within the extended warranty period. This probability is low, so I am leaning toward not buying the extended warranty.

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Repair breakdown:

Appliance	*P*(repair)	Average Repair Cost
washer	19%	$400
dryer	10%	$300
both	3%	$400 + $300 or $700

I decided to do a cost analysis for supporting evidence.

I analyzed the results, along with the average repair costs.

Appliance	*P*(repair)	Cost or Saving
washer	19%	$450 − $400 or $50
dryer	10%	$450 − $300 or $150
neither	68%	$450 − $0 or $450

I subtracted the repair cost for each appliance from the warranty cost to determine whether the warranty would save money.

For each appliance, the warranty costs more than the repair. The warranty is not worth getting.

On average, it would cost $50 less to repair the washer and $150 less to repair the dryer than it would to buy the extended warranty.

both	3%	$450 − $700 or −$250

If both the washer and the dryer need to be repaired, the repair cost would be more than the extended warranty. However, there is only a 3% probability that both appliances will break down.

If both appliances need repair, then, on average, it would cost $250 more than the extended warranty.

There is a 3% chance that the warranty will save my mother $250 and a 97% chance that it will cost her money. She should not buy the warranty because it is more likely to cost money than to save money.

I presented the results of my analysis.

Your Turn

a) Suppose that the average repair cost was $650 for the washer and $490 for the dryer. Should Reid's mother buy the warranty? Justify your decision.

b) Suppose that the probability of both appliances needing repair was 10%. Would your advice for part a) change? Explain.

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EXAMPLE 5 Determining the probability of two events

A car manufacturer keeps a database of all the cars that are available for sale at all the dealerships in Western Canada. For model A, the database reports that 43% have heated leather seats, 36% have a sunroof, and 49% have neither. Determine the probability of a model A car at a dealership having both heated leather seats and a sunroof.

Katherine's Solution

Let A represent the universal set of all model A cars.
Let L represent model A cars with heated leather seats.
Let S represent model A cars with a sunroof.

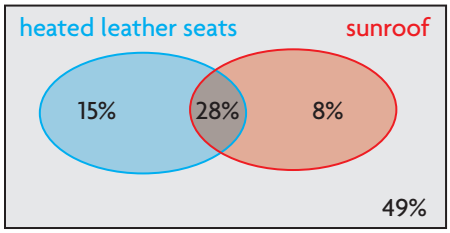
I defined the universal set A and subsets L and S.

I knew that the complement of having neither feature on the car would be having at least one of the two features.

$P(L \cup S) = 100\% - 49\%$
$P(L \cup S) = 51\%$

I used the probability formula for non-mutually exclusive events. I substituted the probabilities I knew and solved for $P(L \cap S)$.

$P(L \cup S) = P(L) + P(S) - P(L \cap S)$
$51\% = 43\% + 36\% - P(L \cap S)$
$28\% = P(L \cap S)$



I drew a Venn diagram to visualize the probabilities in the problem.

The probability of a model A car at a dealership having both heated leather seats and a sunroof is 28%.

Your Turn

The database also reports that 56% of the available model B cars have heated leather seats, 49% have a sunroof, and 27% have neither. What is the probability of a model B car at a dealership having a sunroof but no heated leather seats?

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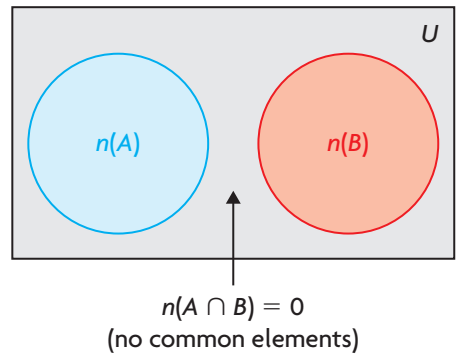
In Summary

Key Ideas

- You can represent the favourable outcomes of two **mutually exclusive events**, A and B, as two disjoint sets.

  You can represent the probability that either A or B will occur by the following formula:

  $$P(A \cup B) = P(A) + P(B)$$



$n(A \cap B) = 0$
(no common elements)

- You can represent the favourable outcomes of two **non-mutually exclusive events**, A and B, as two intersecting sets.

  You can represent the probability that either A or B will occur by this formula:

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

  An alternative formula is

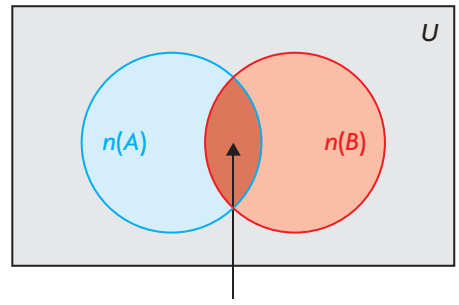
  $$P(A \cup B) = P(A \setminus B) + P(B \setminus A) + P(A \cap B)$$

  When the two events are mutually exclusive, both formulas are equivalent, since

  $$n(A \cap B) = 0$$

  which results in

  $$P(A \cap B) = 0$$



$n(A \cap B)$ has been shaded twice

Need to Know

- You can use the Principle of Inclusion and Exclusion, which is used to count the elements in the union of two sets, to determine the probability of non-mutually exclusive events.

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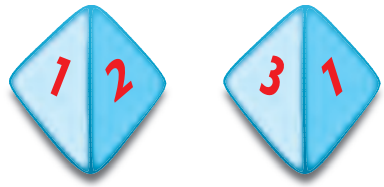
CHECK Your Understanding

1. Zach is playing a board game. He must roll two four-sided dice, numbered 1 to 4. He can move if he rolls a sum of 2 or a sum of 8.



a) Use A and B to represent the two events that will allow Zach to move. Then draw a Venn diagram to illustrate A and B.
b) Are A and B mutually exclusive or not mutually exclusive?
c) Determine the probability that Zach will roll a sum of 2 or a sum of 8.
d) Determine the probability that Zach will roll doubles or a sum of 6.

2. Pearl is about to draw a card at random from a standard deck of 52 playing cards. If she draws a face card or a spade, she will win a point.
a) Draw a Venn diagram to represent the two events.
b) Are the events mutually exclusive?
c) Determine the probability of drawing a face card or a spade.

3. The probability that Maria will go to the gym on Saturday is 0.75. The probability that she will go shopping on Saturday is 0.4. The probability that she will do neither is 0.2.
a) Draw a Venn diagram to represent the two events.
b) Are the two events mutually exclusive?
c) Determine the probability that Maria will do at least one of these activities on Saturday.

PRACTISING

4. For each of the following, state whether the events are mutually exclusive. Explain your reasoning.
a) Selecting a prime number or selecting an even number from a set of 15 balls, numbered 1 to 15
b) Rolling a sum of 10 or a sum of 7 with a pair of six-sided dice, numbered 1 to 6
c) Walking to school or getting a ride to school

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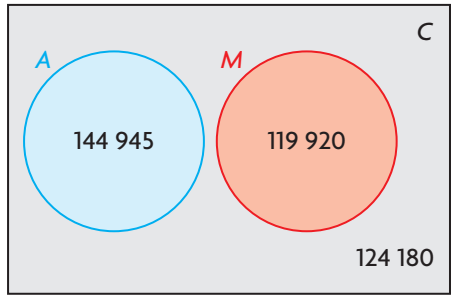
5. The following Venn diagram shows the declared population of Métis in Canada, where
$A = \{\text{Métis in Alberta and British Columbia}\}$,
$M = \{\text{Métis in Manitoba and Saskatchewan}\}$, and
$C = \{\text{Métis in Canada}\}$.



a) Determine the probability that a person who is Métis lives in Alberta or British Columbia.
b) Determine the probability that a person who is Métis lives in Manitoba or Saskatchewan.
c) Does $P(A \cup M) = P(A) + P(M)$ in this situation? Explain.
d) Determine the odds in favour of a person who is Métis living in one of the four Western provinces.

6. Tanya plays the balloon pop game at a carnival. There are 40 balloons, with the name of a prize inside each balloon. The prizes are 8 stuffed bears, 5 toy trucks, 16 decks of cards, 7 yo-yos, and 4 giant stuffed dogs. Tanya pops a balloon with a dart. Determine the odds in favour of her winning either a stuffed dog or a stuffed bear.

7. Edward rolls two regular six-sided dice. Determine the odds against each event below.
a) The sum is 5 or 9.
b) Both dice are even numbers, or the sum is 8.

8. The probability that John will study on Friday night is 0.4.
The probability that he will play video games on Friday night is 0.6.
The probability that he will do at least one of these activities is 0.8.
a) Determine the probability that he will do both activities.
b) Are these events mutually exclusive? Explain how you know.

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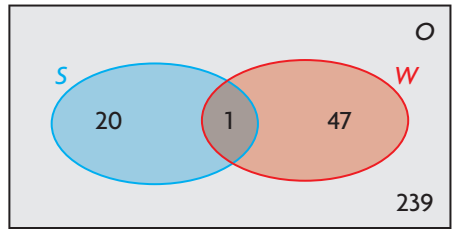
9. The following Venn diagram shows the number of Canadian athletes who have won medals at the Olympics from 1996 to 2010. In the diagram below,

S = {athletes who have won two or more medals at the Summer Olympics},

W = {athletes who have won two or more medals at the Winter Olympics}, and

O = {athletes who have won at least one Olympic medal}.



a) Are the two events (winning two or more medals at the Summer Olympics and winning two or more medals at the Winter Olympics) mutually exclusive? Explain.

b) A Canadian athlete who won a medal at the Summer Olympics from 1996 to 2010 is selected at random. Determine the odds in favour of this athlete having won two or more medals.

c) A Canadian athlete who won a medal at the Summer Olympics or the Winter Olympics from 1996 to 2010 is selected at random. Determine the odds in favour of this athlete having won two or more medals.

Olympian Clara Hughes is the only Canadian who has won medals at both the Summer Olympics and Winter Olympics.

10. Create a problem that involves determining the probability of two mutually exclusive events. Give your problem to a classmate to solve.

11. Create a problem that involves determining the probability of two non-mutually exclusive events. Give your problem to a classmate to solve.

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12. The probability of a 65-year-old person wearing glasses is 76%. The probability of a 65-year-old person having some hearing loss is 68%. Suppose that 10% of 65-year-olds who have a hearing loss do not wear glasses. A 65-year-old person is selected at random.

a) Determine the probability of this person wearing glasses and not having a hearing loss.

b) Determine the probability of this person not wearing glasses and not having a hearing loss.

13. Suppose that you are about to draw a single card, at random, from a standard deck of 52 playing cards. Determine the probability of each event below.

a) You draw an 8 or a king.

b) You draw a red card or a face card.

14. An Ipsos survey reported that 37% of Prairie households have one or more dogs, 31% have one or more cats, and 47% have neither dogs nor cats. Suppose that a Prairie household is selected at random. Determine each probability.

a) There are cats or dogs in the household.

b) There are cats but no dogs in the household.

c) There are dogs but no cats in the household.

15. On Sunday, the weather forecaster says that there is a 60% chance of snow on Monday and a 40% chance of snow on Tuesday. The forecaster also says that there is a 20% chance of snow on both Monday and Tuesday. Determine the probability that there will be snow on Monday or on Tuesday.

16. Suppose that you are the analyst for a computer manufacturer. Today, you are testing a new computer to see whether it needs a surge protector.

- Your tests show that a spike in voltage has a 0.15% chance of damaging the computer's power supply, a 0.30% chance of damaging other components, and a 0.1% chance of damaging the power supply and other components.
- A surge protector is needed if the probability of a spike in voltage damaging a computer exceeds 0.5%.

Would you recommend that this computer should have a surge protector?

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17. Consider this table:

Blood Type	Percent of Canadians with This Blood Type (%)
O+	38
O−	7
A+	34
A−	6
B+	9
B−	2
AB+	3
AB−	1

a) Determine the probability that a randomly selected Canadian will have each type of blood.
i) type O **ii)** a negative blood type **iii)** type A or B

b) A person with type A− blood can donate blood to someone with type A+, A−, AB+, or AB− blood. Suppose that Dani has type A− blood. Determine the probability that she can donate blood to the next person who needs a transfusion.

c) Assume that your local blood bank has a blood supply in the same proportions as shown in the table. A person with type AB− blood can receive type AB−, A−, B−, or O− blood. Richard has type AB− blood. Determine the probability that the most recent donation to the blood bank will be effective for Richard's transfusion.

Closing

18. Describe how determining the probability of two mutually exclusive events is different from determining the probability of two events that are not mutually exclusive. Give an example.

Extending

19. A school newspaper reports on the students' taste in music.
- 20% like only rock.
- 13% like only blues.
- 30% like only rap.
- 10% like rock and blues, but not rap.
- 14% like rock and rap, but not blues.

Determine the probability that a randomly selected student will either like all three types of music or like blues and rap, but not rock.

20. Consider three events, represented by A, B, and C. For each situation, determine a formula for $P(A \cup B \cup C)$.
a) A, B, and C are mutually exclusive.
b) A, B, and C are not mutually exclusive.

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Applying Problem-Solving Strategies

The Monty Hall Puzzle

Monty Hall was born in 1921 in Winnipeg, Manitoba. He hosted the popular television game show *Let's Make a Deal* from 1963 to 1976. (A new version of the show aired in 2009.) If you were a contestant on the show, Monty would show you three doors:

- One door concealed a joke prize, such as a beat-up car or a donkey.
- One door concealed a small prize, such as a new vacuum cleaner.
- One door concealed a grand prize, such as expensive jewellery, a luxury car, or a trip.

You would choose one door. Then Monty would open a door you had not chosen to reveal either the joke prize or the small prize. At this point, you could either stay with your original choice or switch to the other door. You would win the prize behind that door.

When Monty asked if you wanted to switch, should you stay with your original choice or switch?

Analyzing the Puzzle

A. Predict whether switching your choice of door would affect your chance of winning the grand prize.

B. Determine whether your prediction was correct, using a tree diagram or an experimental model. Provide your reasoning to support your representation.

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3.5 Conditional Probability

YOU WILL NEED

- calculator

EXPLORE...

- Jackie plays on a volleyball team called the Giants. The Giants are in a round-robin tournament with five other teams. The teams that they will play against will be selected at random. Determine the probability that their first game will be against the Clippers and their second game will be against the Maroons.

GOAL

Understand and solve problems that involve dependent events.

LEARN ABOUT the Math

A computer manufacturer knows that, in a box of 100 chips, 3 will be defective. Jocelyn will draw 2 chips, at random, from a box of 100 chips.

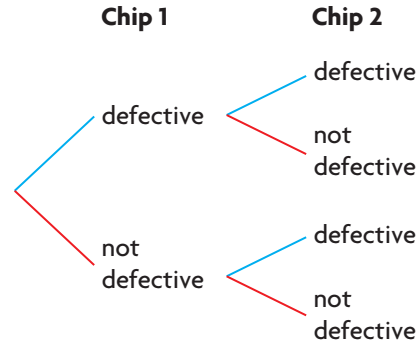
? What is the probability that both of the chips will be defective?

EXAMPLE 1 Calculating the probability of two events

Determine the probability that Jocelyn will draw 2 defective chips.

Jocelyn's Solution

I drew a tree diagram to represent the ways that I could draw 2 computer chips from the box.



There are 4 permutations: both are defective; the first is defective and the second is not defective; the first is not defective and the second is defective; and both are not defective. I am concerned only with the situation where both are defective.

Let A represent the event that the first chip I draw will be defective. Let B represent the event that the second chip I draw will be defective.

I defined the two **dependent events** in this situation.

There are 3 defective chips in the box of 100 chips.

$$P(A) = \frac{3}{100}$$

I determined the probability that the first chip I draw will be defective.

If the first chip I do draw is defective, then the box now holds 99 chips, 2 of which are defective.

$$P(B \mid A) = \frac{2}{99}$$

I determined the **conditional probability** that the second chip I draw will be defective.

dependent events

Events whose outcomes are affected by each other; for example, if two cards are drawn from a deck without replacement, the outcome of the second event depends on the outcome of the first event (the first card drawn).

conditional probability

The probability of an event occurring given that another event has already occurred.

Communication Tip

$P(B \mid A)$ is the notation for a conditional probability. It is read "the probability that event B will occur, given that event A has already occurred."

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$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(A \text{ and } B) = \left(\frac{3}{100}\right)\left(\frac{2}{99}\right)$$

$$P(A \text{ and } B) = \frac{6}{9900} \text{ or } \frac{1}{1650}$$

There is one chance in 1650 that both of the chips I draw will be defective.

I needed to determine the probability that I will draw 2 defective chips.

I knew that you determine the probability of two independent events occurring by multiplying their probabilities. I reasoned that I could determine the probability of two dependent events occurring in the same way. The difference is that one of the probabilities will be conditional if the events are dependent.

Verify:

$$P(A \text{ and } B) = \frac{3 \cdot 2}{100 \cdot 99}$$

$$P(A \text{ and } B) = \frac{6}{9900} \text{ or } \frac{1}{1650}$$

I got the same answer.

I verified my conjecture using the Fundamental Counting Principle. The probability that I will randomly select 2 defective chips, without replacement, from the box is the number of ways of drawing the 2 defective chips ($3 \cdot 2$) divided by the number of ways of drawing any 2 chips ($100 \cdot 99$).

Reflecting

A. In this example, how does the probability that the first event will occur affect the probability that the second event will occur? Explain.

B. Suppose that Jocelyn replaced the first chip before drawing the second chip. Would the probability of the second chip being defective remain the same? Explain.

C. When determining the probability of drawing two defective chips, Jocelyn did not consider the possibility that the first chip she drew would not be defective. Explain why.

D. Explain why the probability of drawing a defective chip on the second draw is considered a conditional probability.

E. Starting with a tree diagram like Jocelyn's, label each branch with its probability. Determine the probability of drawing each permutation of defective and not defective chips, then add these probabilities. What does the sum imply?

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APPLY the Math

EXAMPLE 2 Calculating the conditional probability of a pair of dependent events

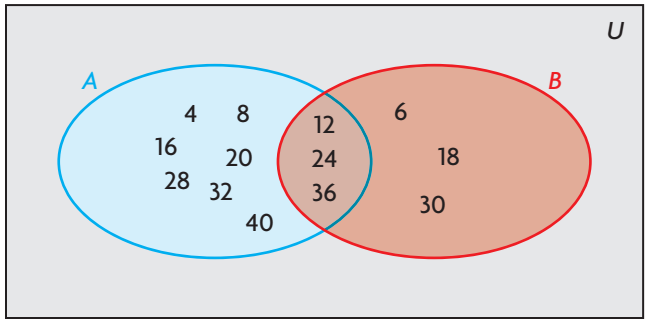
Nathan asks Riel to choose a number between 1 and 40 and then say one fact about the number. Riel says that the number he chose is a multiple of 4. Determine the probability that the number is also a multiple of 6, using each method below.

a) A Venn diagram **b)** A formula

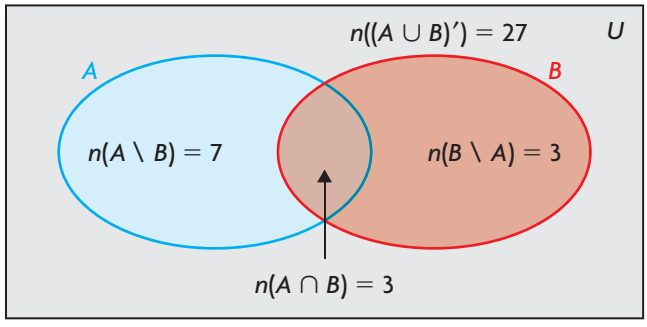
Nathan's Solution

a) Let U = {all natural numbers from 1 to 40}.
Let A = {multiples of 4 from 1 to 40}.
Let B = {multiples of 6 from 1 to 40}.

I defined the universal set and the events.



I drew a Venn diagram to show the sets. I wrote all the numbers that Riel could have chosen in my diagram.



I redrew the Venn diagram to show the number of elements in each area.

The universal set has 40 elements, and $A \cup B$ has 13 elements.
Therefore, $(A \cup B)'$ must have $40 - 13$ or 27 elements.
Riel could have chosen one of 10 numbers. Only three of these numbers are multiples of 4 and 6.
Since the probability that Riel's number is a multiple of 6 is $\frac{3}{10}$,

$$P(B \mid A) = \frac{3}{10}$$

I knew that 10 of the numbers from 1 to 40 are multiples of 4. This is $n(A)$, which is the total number of outcomes for the conditional probability.

Three of these numbers are also multiples of 6. This is $n(A \cap B)$, which is the number of favourable outcomes for the conditional probability.

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b) $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

I wrote the formula for determining the probability of two dependent events.

From my Venn diagram, I saw that the area of overlap, or the intersection, is (A and B).

I substituted $(A \cap B)$ into the formula.

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$\frac{P(A \cap B)}{P(A)} = P(B \mid A)$$

I solved for $P(B \mid A)$ to develop a formula for determining a conditional probability.

I determined the values of $P(A)$ and $P(A \cap B)$ using my Venn diagram.

$$P(A) = \frac{10}{40}$$

$$P(A \cap B) = \frac{3}{40}$$

$$\frac{\frac{3}{40}}{\frac{10}{40}} = P(B \mid A)$$

I substituted these values into my formula to determine $P(B \mid A)$.

$$\frac{3}{10} = P(B \mid A)$$

This is the same answer that I got from the Venn diagram.

This makes sense because there are 3 numbers in $(A \cap B)$ and 10 numbers in set A. Since $\frac{3}{40}$ and $\frac{10}{40}$ have the same denominator, I got $\frac{3}{10}$ when I divided these probabilities.

Your Turn

Nathan chose another number from 1 to 40 and told Riel that it is a multiple of 6. Determine the probability that this number is also a multiple of 4.

EXAMPLE 3 Solving a conditional probability problem

According to a survey, 91% of Canadians own a cellphone. Of these people, 42% have a smartphone. Determine, to the nearest percent, the probability that any Canadian you met during the month in which the survey was conducted would have a smartphone.

Tara's Solution

I let C represent owning a cellphone.
I let S represent owning a smartphone.

I defined the events.

$$P(C) = 0.91$$
$$P(S \mid C) = 0.42$$

I wrote the probability that a Canadian owns a cellphone and the conditional probability that a Canadian owns a smartphone, given that she or he owns a cellphone.

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$$P(S \mid C) = \frac{P(S \cap C)}{P(C)}$$

$$0.42 = \frac{P(S \cap C)}{0.91}$$

$$(0.42)(0.91) = P(S \cap C)$$

$$0.3822 = P(S \cap C)$$

I substituted the known probabilities into the formula for conditional probability.

I solved for $P(S \cap C)$.

Smartphones are a subset of cellphones, so it makes sense that $P(S)$ is equal to $P(S \cap C)$.

$$P(S) = P(S \cap C)$$

$$P(S) = 38.22\%$$

To the nearest whole number, the probability that any Canadian I met in that month would have a smartphone is 38%.

Your Turn

a) Determine, to the nearest whole number, the probability that any Canadian you met in that month would have a cellphone but not a smartphone.

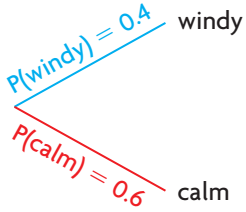
b) How could you represent this probability in Tara's Venn diagram?

EXAMPLE 4 Making predictions that involve dependent events

Hillary is the coach of a junior ultimate team. Based on the team's record, it has a 60% chance of winning on calm days and a 70% chance of winning on windy days. Tomorrow, there is a 40% chance of high winds. There are no ties in ultimate. What is the probability that Hillary's team will win tomorrow?

Hillary's Solution

$P(\text{windy})$ is 40%, so $P(\text{calm})$ is 100% − 40% or 60%.



I determined the probability of a calm day tomorrow.

Then I drew the first branches of my tree diagram.

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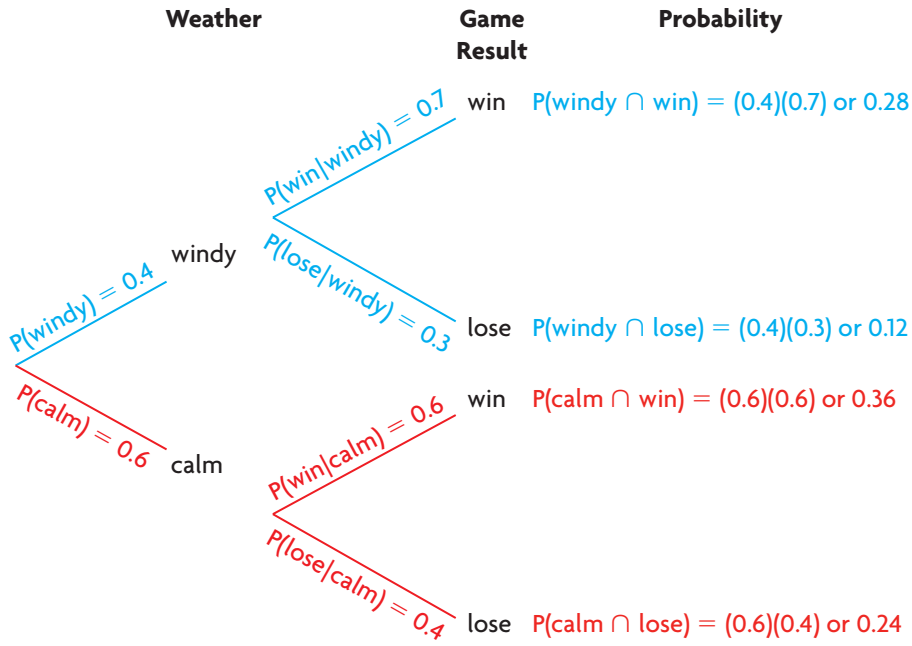
$P(\text{win} \mid \text{windy}) = 70\%$
$P(\text{lose} \mid \text{windy}) = 100\% - 70\%$ or 30%
$P(\text{win} \mid \text{calm}) = 60\%$
$P(\text{lose} \mid \text{calm}) = 100\% - 60\%$ or 40%

I determined the probabilities of losing on windy days and calm days. Since there are no ties in ultimate, these events are complementary.



I completed my tree diagram by multiplying the probabilities along each branch. I knew that I could do this because the events on each branch are pairs of dependent events.

$P(\text{win}) = P(\text{windy} \cap \text{win}) + P(\text{calm} \cap \text{win})$
$P(\text{win}) = 0.28 + 0.36$
$P(\text{win}) = 0.64$
The probability that we will win tomorrow is 64%.

To determine the probability that we will win tomorrow, I added both probabilities that would result in a win. I could do this because the two events (windy and win, and calm and win) are mutually exclusive. Only one of these events will occur.

Your Turn

a) Explain how you can check quickly to determine if Hillary made any errors in her tree diagram.

b) Determine the probability that Hillary's team will lose tomorrow. Verify your solution using another method.

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In Summary

Key Ideas

- If the probability of one event depends on the probability of another event, then these events are called **dependent events**. For example, drawing a heart from a standard deck of 52 playing cards and then drawing another heart from the same deck without replacing the first card are dependent events.
- If event B depends on event A occurring, then the **conditional probability** that event B will occur, given that event A has occurred, can be represented as follows:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

Need to Know

- If event B depends on event A occurring, then the probability that both events will occur can be represented as follows:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

- A tree diagram is often useful for modelling problems that involve dependent events.
- Drawing an item and then drawing another item, without replacing the first item, results in a pair of dependent events.

CHECK Your Understanding

1. Austin rolls a regular six-sided red die and a regular six-sided black die. If the red die lands on 4 and the sum of the two dice is greater than 7, Austin wins a point.

a) Are the two events dependent or independent?

b) Determine the probability that Austin will win a point.

2. Valeria draws a card from a well-shuffled standard deck of 52 playing cards. Then she draws another card from the deck without replacing the first card.

a) Are these two events dependent or independent?

b) Determine the probability that both cards are diamonds.

3. Valeria draws a card from a well-shuffled standard deck of 52 playing cards. Then she puts the card back in the deck, shuffles again, and draws another card from the deck.

a) Are these two events dependent or independent?

b) Determine the probability that both cards are diamonds.

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PRACTISING

4. Lexie has six identical black socks and eight identical white socks loose in her drawer. She pulls out one sock at random and then another sock, without replacing the first sock.

a) Determine the probability of each event below.

i) She pulls out a pair of black socks.

ii) She pulls out a pair of white socks.

iii) She pulls out a matched pair of socks; that is, either both are black or both are white.

b) If Lexie randomly pulled out both socks at the same time, would your answers for part a) change? Explain.

5. There are 80 males and 110 females in the graduating class in a Kelowna school. Of these students, 30 males and 50 females plan to attend the University of British Columbia (UBC) next year.

a) Determine the probability that a randomly selected student plans to attend UBC.

b) A randomly selected student plans to attend UBC. Determine the probability that the selected student is female.

6. Each day, Melissa's math teacher gives the class a warm-up question. It is a true-false question 30% of the time and a multiple-choice question 70% of the time. Melissa gets 60% of the true-false questions correct, and 80% of the multiple-choice questions correct. Melissa answers today's question correctly. What is the probability that it was a multiple-choice question?

7. Skye has four loonies, three toonies, and five quarters in his pocket. He needs two loonies for a parking meter. He reaches into his pocket and pulls out two coins at random. Determine the probability that both coins are loonies.

8. Anita remembers to set her alarm clock 62% of the time. When she does remember to set her alarm clock, the probability that she will be late for school is 0.20. When she does not remember to set it, the probability that she will be late for school is 0.70. Anita was late today. What is the probability that she remembered to set her alarm clock?

9. Ian likes to go for daily jogs with his dog, Oliver. If the weather is nice, he is 85% likely to jog for 8 km. If the weather is rainy, he is only 40% likely to jog for 8 km. The weather forecast for tomorrow indicates a 30% chance of rain. Determine the probability that Ian will jog for 8 km.

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10. Cellphone users in Mapletown were surveyed about their phone plan, with these results:

- 70% of all users have call display.
- 40% of users have a data plan.
- 75% users with a data plan also have call display.

A cellphone user in Mapletown, who is selected at random, has call display. Determine the probability that this person also has a data plan.

11. Cole surveyed 10 students in his Grade 12 class about their lunch break on school days. He asked them to base their answers to the following questions on a period of 1 month.

- How often do you have 1 h or less for lunch?
- How often do you have more than 1 h for lunch?
- How often do you go to your local fast-food outlet for lunch, if you only have 1 h or less for lunch?
- How often do you go to your local fast-food outlet for lunch, if you have more than 1 h for lunch?

His results are given below:

A = {1 h or less for lunch}
B = {more than 1 h for lunch}
C = {go to fast food outlet if less than 1 h for lunch}
D = {go to fast food outlet if more than 1 h for lunch}

Event	Number
A	120
B	80
C	40
D	60

Create a conditional probability problem for Cole's data.

12. Decide on a topic that interests you.

a) Create and conduct a short survey, similar to Cole's survey in the previous question. Tabulate your results.

b) Create a conditional probability problem for your data.

13. The probability that a car tire will last for 5 years is 0.8. The probability that a tire will last for 6 years is 0.5. Suppose that your parents' tires have lasted for 5 years. Determine the probability that the tires will last for 6 years.

14. The probability that the windshield wipers on a particular model of car will last for 2 years is 0.7. The probability that they will last for 3 years is 0.6. The wipers on your parents' car have lasted for 2 years. Determine the probability that the wipers will last for 3 years.

15. The probability that a particular pair of badminton shoes will last for 6 months is 0.9. The probability that the shoes will last for 1 year is 0.2. Natalie's shoes have lasted for 6 months. Determine the probability that they will last for 1 year.

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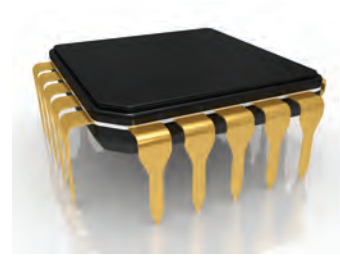
16. Morgan asks Jasmine to choose a number between 20 and 50 and then say one fact about the number. Jasmine says that the number she chose is a multiple of 5. Determine the probability that the number is also a multiple of 3, using each method below.
- **a)** A Venn diagram
- **b)** The formula for conditional probability

17. Recall the opening problem in this lesson:
A computer manufacturer knows that, in a box of 100 chips, 3 will be defective. Jocelyn will draw 2 chips, at random, from a box of 100 chips.
- **a)** Draw a tree diagram to represent this situation.
- **b)** Determine the probability that exactly 1 chip will be defective. Explain what you did.



18. A computer manufacturer knows that, in a box of 150 computer chips, 3 will be defective. Samuel will draw 2 chips, at random, from a box of 150 chips. Determine the probability that Samuel will draw the following:
- **a)** 2 defective chips
- **b)** 2 non-defective chips
- **c)** Exactly 1 defective chip

19. Savannah's soccer team is playing a game tomorrow. Based on the team's record, it has a 50% chance of winning on rainy days and a 60% chance of winning on sunny days. Tomorrow, there is a 30% chance of rain. Savannah's soccer league does not allow ties.
- **a)** Determine the probability that Savannah's team will win tomorrow.
- **b)** Determine the probability that her team will lose tomorrow.

20. Think of two situations in your life in which the probability of one event happening depends on another event happening. Write two problems, one for each of these situations. Also, write the solutions to your problems. Exchange your problems with a classmate. Solve, and then correct, each other's problems. Adjust your problems, if necessary.

Closing

21. Explain the meaning of the formula $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$. Give an example to illustrate your explanation.

Extending

22. A computer manufacturer knows that, in a box of 100 computer chips, 4 will be defective. Caleb will draw 3 chips, at random, from a box of 100 chips. Determine the probability that Caleb will draw the following:
- **a)** 3 defective chips
- **b)** 3 non-defective chips
- **c)** More defective chips than non-defective chips

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3.6 Independent Events

YOU WILL NEED

- calculator

EXPLORE...

- The Fortin family has two children. Cam determines the probability that the family has two girls. Rushanna determines the probability that the family has two girls, given that one child is a girl. How are these probabilities similar, and how are they different?

GOAL

Understand and solve problems that involve independent events.

INVESTIGATE the Math

Anne and Abby each have 19 marbles: 11 red and 8 blue. Anne places 7 red marbles and 3 blue marbles in bag 1. She places the rest of her marbles in bag 2. Abby places all of her marbles in bag 3. Anne then draws one marble from bag 1 and one marble from bag 2. Abby draws two marbles from bag 3, without replacement.

? Are both girls equally likely to draw two red marbles?

A. Let A represent Anne drawing a red marble from bag 1.
Let B represent Anne drawing a red marble from bag 2.
Are A and B independent or dependent events? Explain.

B. Let C represent Abby drawing a red marble from bag 3 on her first draw. Let D represent Abby drawing a red marble from bag 3 on her second draw. Are C and D independent or dependent events? Explain.

C. Determine $P(A)$ and $P(B)$.

D. Determine $P(A \cap B)$ to three decimal places.

E. Determine $P(C)$ and $P(D \mid C)$, the probability that Abby will draw a red marble from bag 3 on the second draw, given that the first marble she drew was red.

F. Determine $P(C \cap D)$ to three decimal places.

G. Are both girls equally likely to draw two red marbles? Explain.

Reflecting

H. Explain how you can tell if two events are independent or dependent.

I. Suppose that Abby were to draw both of her marbles together, instead of drawing one after the other. Would this affect the probability of Abby drawing two red marbles? Justify your answer.

J. Naveen used the following formula to determine the probability of Anne drawing two red marbles:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

Would this formula give the correct answer? Explain.

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APPLY the Math

EXAMPLE 1 Determining probabilities of independent events

Mokhtar and Chantelle are playing a die and coin game. Each turn consists of rolling a regular die and tossing a coin. Points are awarded for rolling a 6 on the die and/or tossing heads with the coin:

- 1 point for either outcome
- 3 points for both outcomes
- 0 points for neither outcome

Players alternate turns. The first player who gets 10 points wins.

a) Determine the probability that Mokhtar will get 1, 3, or 0 points on his first turn.

b) Verify your results for part a). Explain what you did.

Chantelle's Solution

a) Let S represent rolling a 6 on the die.
Let H represent tossing heads with the coin.

$$P(S) = \frac{1}{6} \qquad P(H) = \frac{1}{2}$$

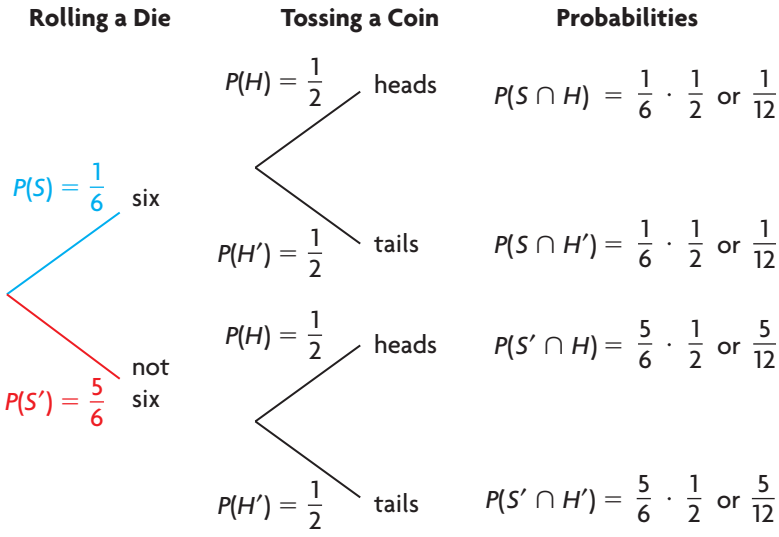
I defined events S and H. These events are independent. The probability of tossing heads with the coin is not affected by the probability of rolling 6 on the die.

$$P(S') = 1 - \frac{1}{6} \qquad P(H') = 1 - \frac{1}{2}$$

$$P(S') = \frac{5}{6} \qquad P(H') = \frac{1}{2}$$

I determined the probability that Mokhtar will not get either outcome when rolling the die and tossing the coin. These events are complementary.

I created a tree diagram showing the probabilities associated with the events of rolling a six, S, on a die and tossing heads with a coin, H, along with the probabilities of their complements, $S'$and H'.



I determined the probabilities of each pair of events occurring on the tree diagram by multiplying.

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P(scores 3 points): $P(S \cap H) = \frac{1}{12}$

Mokhtar gets 3 points when he rolls a 6 and tosses heads.

P(scores 0 points): $P(S' \cap H') = \frac{5}{12}$

He gets 0 points when he does not roll a 6 and does not toss heads.

P(scores 1 point): $P(S' \cap H) \cup P(S \cap H') = \frac{5}{12} + \frac{1}{12}$

$P(S' \cap H) \cup P(S \cap H') = \frac{6}{12}$ or $\frac{1}{2}$

He gets 1 point when he either rolls a 6 or tosses heads

He has a $\frac{1}{12}$ chance of scoring 3 points, a $\frac{1}{2}$ chance of scoring 1 point, and a $\frac{5}{12}$ chance of scoring 0 points.

b) Verify:

Total probability $= \frac{1}{12} + \frac{5}{12} + \frac{6}{12}$

Total probability $= \frac{12}{12}$ or 1

I verified my solution by adding the three probabilities. The sum is 1, so I knew that my answer is correct.

Your Turn

a) Are you more likely to get points or not get points on each turn? Explain.
b) Would the probabilities determined for Mokhtar's first turn change for his next turn? Explain.

EXAMPLE 2 Solving a problem that involves independent events using a graphic organizer

All 1000 tickets for a charity raffle have been sold and placed in a drum. There will be two draws. The first draw will be for the grand prize, and the second draw will be for the consolation prize. After each draw, the winning ticket will be returned to the drum so that it might be drawn again. Max has bought five tickets. Determine the probability, to a tenth of a percent, that he will win at least one prize.

Max's Solution: Using complements

Let X represent winning the grand prize.
Let Y represent winning the consolation prize.
Let Z represent winning at least one prize.

I defined events X, Y, and Z.

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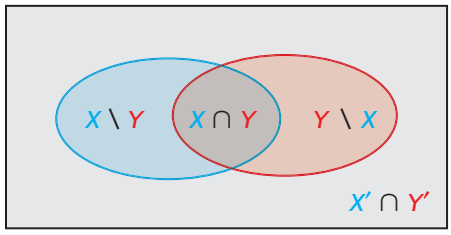
$$P(X) = \frac{5}{1000} \text{ or } \frac{1}{200}$$
$$P(Y) = \frac{5}{1000} \text{ or } \frac{1}{200}$$

The events are independent.

Since the winning ticket will be replaced after the first draw, the probability of winning either prize is the same.

In other words, winning the consolation prize is not affected by winning the grand prize.



I drew a Venn diagram to represent the situation.

The event of winning at least one prize, Z, is represented by being in any area of the two ovals in the Venn diagram.

$P(Z)$, the probability of winning at least one prize, is the complement of winning no prizes:

$$P(Z) = 1 - P(X' \cap Y')$$

I had to determine the probability of winning the grand prize or the consolation prize.

The only other possibility is winning no prizes.

$$P(X') = 1 - \frac{1}{200} \text{ or } \frac{199}{200}$$
$$P(Y') = 1 - \frac{1}{200} \text{ or } \frac{199}{200}$$
$$P(X' \cap Y') = P(X') \cdot P(Y')$$
$$P(X' \cap Y') = \frac{199}{200} \cdot \frac{199}{200}$$
$$P(X' \cap Y') = \frac{39\,601}{40\,000}$$

I determined the probability of not winning the grand prize, the probability of not winning the consolation prize, and the probability of winning neither prize.

$$P(Z) = 1 - P(X' \cap Y')$$
$$P(Z) = 1 - \frac{39\,601}{40\,000}$$
$$P(Z) = \frac{40\,000}{40\,000} - \frac{39\,601}{40\,000}$$
$$P(Z) = \frac{399}{40\,000} \text{ or } 0.009\,975$$

I determined the probability of winning at least one prize.

The probability Max will win at least one prize is about 1.0%.

I rounded to the nearest tenth of a percent.

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Melissa's Solution: Using a tree diagram

Let X represent winning the grand prize.
Let Y represent winning the consolation prize.
Let Z represent winning at least one prize.

I defined events X, Y, and Z.

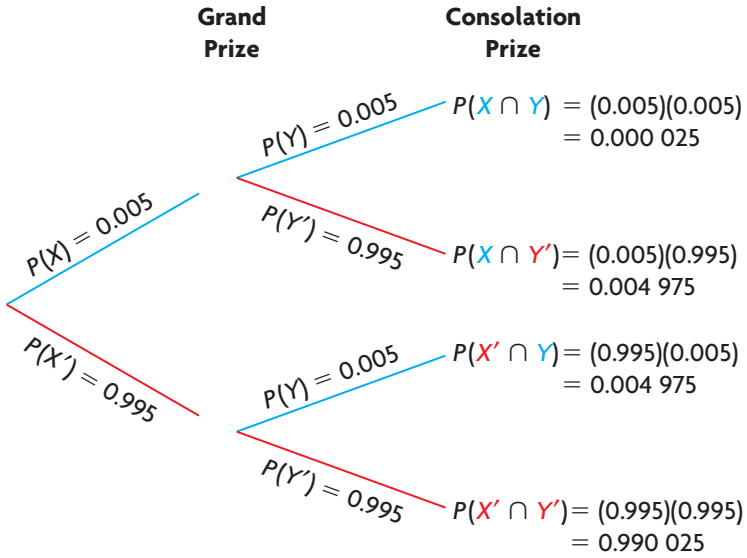
$$P(X) = \frac{5}{1000} \text{ or } 0.005$$

$$P(Y) = \frac{5}{1000} \text{ or } 0.005$$

I determined the probability of winning the grand prize and the probability of winning the consolation prize.

$$P(X') = 1 - P(X) \qquad P(Y') = 1 - P(Y)$$
$$P(X') = 1 - 0.005 \qquad P(Y') = 1 - 0.005$$
$$P(X') = 0.995 \qquad P(Y') = 0.995$$

I also determined the complement of these events: the probability of not winning the grand prize and the probability of not winning the consolation prize.



I drew a tree diagram to list all the possible outcomes. The blue branches represent winning a prize, and the red branches represent not winning a prize. I determined the probability of each pair of events by multiplying the individual probabilities.

Based on my tree diagram, Max could win two prizes in one way.

$$P(\text{win 2 prizes}) = P(X \cap Y)$$
$$P(\text{win 2 prizes}) = 0.000\ 025$$

Max could win one prize in two ways.

$$P(\text{win 1 prize}) = P(X \cap Y') \text{ or } P(X' \cap Y)$$
$$P(\text{win 1 prize}) = P(X \cap Y') \cup P(X' \cap Y)$$
$$P(\text{win 1 prize}) = 0.004\ 975 + 0.004\ 975$$
$$P(\text{win 1 prize}) = 0.009\ 95$$

These probabilities are equal and can be added together, since the two events (only winning the grand prize or only winning the consolation prize) are mutually exclusive.

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Winning at least one prize means that Max would win exactly one prize or both prizes.

$P(Z) = P(\text{win 1 prize}) \text{ or } P(\text{win 2 prizes})$
$P(Z) = P(\text{win 1 prize}) \cup P(\text{win 2 prizes})$
$P(Z) = P(\text{win 1 prize}) + P(\text{win 2 prizes})$
$P(Z) = 0.009\,95 + 0.000\,025$
$P(Z) = 0.009\,975$

These probabilities are different and can be added together, since the two events (winning one prize and winning two prizes) are mutually exclusive.

The probability Max will win at least one prize is 1.0%.

I rounded to the nearest tenth of a percent.

Verify:
Total probability = $P(\text{win at least one}) + P(\text{win none})$
Total probability = $0.009\,975 + 0.990\,025$
Total probability = 1

I verified my calculations by adding the probabilities. The sum is 1, so I knew that my answer is correct.

Your Turn

Suppose that the rules for the raffle are changed, so the first ticket drawn is not returned to the drum before the second draw.

a) Are the events winning the grand prize and winning the consolation prize dependent or independent?

b) Do you think Max's probability of winning at least one prize will be greater or less than before? Justify your answer.

In Summary

Key Ideas

- If the probability of event B does not depend on the probability of event A occurring, then these events are called independent events. For example, tossing tails with a coin and drawing the ace of spades from a standard deck of 52 playing cards are independent events.
- The probability that two independent events, A and B, will both occur is the product of their individual probabilities:

$$P(A \cap B) = P(A) \cdot P(B)$$

Need to Know

- A tree diagram is often useful for modelling problems that involve independent events.
- Drawing an item and then drawing another item, after replacing the first item, results in a pair of independent events.

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CHECK Your Understanding

1. For each situation, classify the events as either independent or dependent. Justify your classification.

a) A four-colour spinner is spun, and a die is rolled. The first event is spinning red, and the second event is rolling a 2.

b) A red die and a green die are rolled. The first event is rolling a 1 on the red die, and the second event is rolling a 5 on the green die.

c) Two cards are drawn, without being replaced, from a standard deck of 52 playing cards. The first event is drawing a king, and the second event is drawing an ace.

d) There are 30 cards, numbered 1 to 30, in a box. Two cards are drawn, one at a time, with replacement. The first event is drawing a prime number, and the second event is drawing a number that is a multiple of 5.

2. Celeste goes to the gym five days a week. Each day, she does a cardio workout using either a treadmill, an elliptical walker, or a stationary bike. She follows this with a strength workout using either free weights or the weight machines. Celeste randomly chooses which cardio workout and which strength workout to do each day.

a) Are choosing a cardio workout and choosing a strength workout dependent or independent events? Explain.

b) Determine the probability that Celeste will use a stationary bike and free weights the next day she goes to the gym.

3. Ian also goes to the gym five days a week, but he does two different cardio workouts each day. His choices include using a treadmill, a stepper, or an elliptical walker, and running the track.

a) Are the two cardio workouts that Ian chooses dependent or independent events?

b) Determine the probability that the next time Ian goes to the gym he will use the elliptical walker and then run the track.

PRACTISING

4. For each situation described in question 1, determine the probability that both events will occur.

5. a) Suppose that $P(A) = 0.35$, $P(B) = 0.4$, and $P(A \cap B) = 0.12$. Are A and B independent events? Explain.

b) Suppose that $P(Q) = 0.720$, $P(R) = 0.650$, and $P(Q \cap R) = 0.468$. Are Q and R independent events? Explain.

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6. There are two children in the Angel family.

a) Draw a tree diagram that shows all the possible gender combinations for the two children.

b) Determine the probability that both children are boys.

c) Determine the probability that one child is a boy and the other child is a girl.

7. A particular game uses 40 cards from a standard deck of 52 playing cards: the ace to the 10 from the four suits. One card is dealt to each of two players. Determine the probability that the first card dealt is a club and the second card dealt is a heart. Are these events independent or dependent?

8. A die is rolled twice. Determine the probability for the following:

a) The first roll is a 1, and the second roll is a 6.

b) The first roll is greater than 3, and the second roll is even.

c) The first roll is greater than 1, and the second roll is less than 6.

9. Jeremiah is going on a cruise up the Nile. According to the travel brochure, the probability that he will see a camel is $\frac{4}{5}$, and the probability that he will see an ibis is $\frac{3}{4}$. Determine the probability that Jeremiah will see the following:

a) A camel and an ibis

b) Neither a camel nor an ibis

c) Only one of these sights

10. **a)** Design a spinner so that when you toss a coin and spin the spinner, the probability of getting heads and spinning a 6 is $\frac{1}{12}$.

b) Repeat part a) with a probability of $\frac{1}{20}$.

11. Recall Anne and Abby, from the beginning of this lesson. They each have 19 marbles: 11 red and 8 blue. Anne places 7 red marbles and 3 blue marbles in bag 1. She places the rest of her marbles in bag 2. Abby places all of her marbles in bag 3. Anne then draws one marble from bag 1 and one marble from bag 2. Abby draws two marbles from bag 3.

a) Are Anne and Abby equally likely to draw two blue marbles from their bags? Explain.

b) Determine the probability Anne and Abby will both draw one red marble and one blue marble. Explain what you did.

c) Suppose that Anne now has 5 red marbles and 5 blue marbles in each of her two bags, while Abby has 10 red marbles and 10 blue marbles in her one bag. Will Abby still be more likely to draw two red marbles? Explain.

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12. A paper bag contains a mixture of three types of treats: 10 granola bars, 7 fruit bars, and 3 cheese strips. Suppose that you play a game in which a treat is randomly taken from the bag and replaced, and then a second treat is drawn from the bag. You are allowed to keep the second treat only if it was the same type as the treat that was drawn the first time. Determine the probability of each of the following:
- **a)** You will be able to keep a granola bar.
- **b)** You will be able to keep any treat.
- **c)** You will not be able to keep any treat.

13. Tiegan's school is holding a chocolate bar sale. For every case of chocolate bars sold, the seller receives a ticket for a prize draw. Tiegan has sold five cases, so she has five tickets for the draw. At the time of the draw, 100 tickets have been entered. There are two prizes, and the ticket that is drawn for the first prize is returned so it can be drawn for the second prize.
- **a)** Determine the probability that Tiegan will win both prizes.
- **b)** Determine the probability that she will win no prizes.

14. **a)** Create a problem that involves determining the probability of two independent events. Give your problem to a classmate to solve.
- **b)** Modify the problem you created in part a) so that it now involves two dependent events. Give your problem to a classmate to solve.

15. Two single-digit random numbers (0 to 9 inclusive) are selected independently. Determine the probability that their sum is 10.

Closing

16. **a)** Explain why the formula you would use to calculate $P(A \cap B)$ would depend on whether A and B are dependent or independent events.
- **b)** Give an example of how you would calculate $P(A \cap B)$ if A and B were independent events.
- **c)** Give an example of how you would calculate $P(A \cap B)$ if A and B were dependent events.

Extending

17. A particular machine has 100 parts. Over a year, the probability that each part of the machine will fail is 1%. If any part fails, the machine will stop.
- **a)** Determine the probability that the machine will operate continuously for 1 year.
- **b)** Suppose that the probability of each part failing within one year dropped to 0.5%. Determine the probability that the machine will operate continuously for 1 year.
- **c)** Suppose that the probability of the machine operating continuously for 1 year must be 90%. What would the probability of not failing need to be for each part?

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18. In the final series of the Stanley Cup Playoffs, the first team to win four games becomes the National Hockey League (NHL) champion. The Original Six teams in the NHL were the Boston Bruins, the Chicago Blackhawks, the Detroit Red Wings, the Montréal Canadiens, the New York Rangers, and the Toronto Maple Leafs.

a) In 1960, Montréal played Toronto and won the first four games. Suppose that the probability of Montréal winning any single game was 0.65. Determine the probability that Montréal would win the series in four games.

b) In 1967, the NHL added six more teams, including the St. Louis Blues. In 1968, Montréal beat St. Louis, winning four games in a row. The odds in favour of this occurring were 5 : 4. Determine the probability that Montréal would win any single game in the series. What assumptions did you make?

Lorne "Gump" Worsley played for the Montréal Canadiens, starting in 1963. He won Stanley Cups in 1965, 1966, 1968, and 1969.

19. Gavin is taking two tests. The true-false test has five questions. The multiple-choice test has five questions, each with four choices. Gavin did not study for the tests, so he has to guess each answer. A pass is over 50%. Use a tree diagram to determine the probability that Gavin will pass each test.

Math in Action

Modelling with Probabilities

You can use experimental and theoretical probabilities to solve a mystery.

Without watching, have a classmate put some of the balls (or tiles) in a container. Can you deduce how many of each colour are in the container?

- To arrive at your conjecture, you can run two types of experiments:
  - Draw out one ball, replace the ball, and repeat. Record your results.
  - Draw out two balls, replace the balls, and repeat. Record your results.
- Compare the results of your experiments with theoretical probabilities to make your conjecture.
- Which type of experiment seemed to be the most productive?
- Based on what your group and other groups did, how could you make your conjecture more reliable?
- Is any experiment you could run guaranteed to result in a correct conjecture? Explain.

YOU WILL NEED

- several balls or tiles, in two different colours
- container

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3 Chapter Self-Test

1. Cassie and Jacob are playing a game. Six tiles, numbered 1 to 6, are placed face down. Each player selects a tile. If the sum of their two tiles is even, Jacob wins. If the sum is odd, Cassie wins. Is this game fair? Explain.
2. The odds in favour of an Inuit person being able to converse in more than one Aboriginal language are 7 : 3. Determine the probability that an Inuit person can converse in at least two Aboriginal languages.
3. A credit card company randomly generates a temporary three-digit pass code for cardholders. Braydon is expecting his credit card to arrive in the mail. Determine the probability that her pass code is made up of three different even digits.
4. In the card game called Crazy Eights, players are dealt 8 cards from a standard deck of 52 playing cards. Determine the probability that a hand will consist of 4 hearts and 4 spades.
5. Kaylee plays the balloon pop game at a carnival. There are 50 balloons, with the name of a prize inside each balloon. The prizes are 10 stuffed bears, 4 toy trucks, 20 stuffed rabbits, 6 yo-yos, and 10 giant stuffed cats. Kaylee pops a balloon with a dart. Determine the probability that she will win either a toy truck or a yo-yo.
6. Hans tosses a coin and then draws a card from a standard deck of 52 playing cards. Determine the probability that he will toss a head and draw an 8.
7. Jarrod likes to play basketball. Based on past games, the probability that he will make a free throw is 70%. If he has been awarded two free throws, what is the probability of each of the following?
   a) He will make both shots.
   b) He will miss both shots.
   c) He will make one shot.
   d) He will make at least one shot.
8. A euchre deck consists of 24 cards: the 9, 10, jack, queen, king, and ace of all four suits. George draws two cards from a well-shuffled euchre deck. Determine the probability that both cards are hearts.
9. Miguel remembers to set his alarm clock 72% of the time. When he does remember to set his alarm clock, the probability that he will be late for school is 0.10. When he does not remember to set it, the probability that he will be late for school is 0.70. Miguel was late today. Determine the probability that he remembered to set his alarm clock.

***WHAT** DO You Think Now?* Revisit **What Do You Think?** on page 139. How have your answers and explanations changed?

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3 Chapter Review

FREQUENTLY ASKED Questions

Q: How can you determine if two events are mutually exclusive? Then how can you determine the probability that either of these events will occur?

A: Two events, A and B, are mutually exclusive if they cannot happen at the same time. For example, a coin can land heads or tails, but not heads and tails.

If two events are mutually exclusive, $n(A \cap B) = 0$. The favourable outcomes are represented by a pair of disjoint sets:

$$n(A \cup B) = n(A) + n(B)$$

To determine the probability that either of two mutually exclusive events will occur, add the probabilities of the two events:

$$P(A \cup B) = P(A) + P(B)$$

For example, consider these sets:

- the universal set
  $U = \{\text{natural numbers from 1 to 36, inclusive}\}$
- set A, the odd multiples of 3
  $A = \{3, 9, 15, 21, 27, 33\}$
- set B, the multiples of 4
  $B = \{4, 8, 12, 16, 20, 24, 28, 32, 36\}$

To determine the probability that a natural number chosen at random is either an odd multiple of 3 or a multiple of 4, use the following formula:

$$P(A \cup B) = P(A) + P(B)$$

Since A and B are mutually exclusive, $(A \cap B) = \{\ \}$.

$$P(A \cup B) = \frac{6}{36} + \frac{9}{36}$$

$$P(A \cup B) = \frac{15}{36}$$

The probability that a natural number chosen at random from the numbers 1 to 36 is either an odd multiple of 3 or a multiple of 4 is $\frac{15}{36}$.

Study *Aid*

- See Lesson 3.4, Examples 1, 3, 4, and 5.
- Try Chapter Review Questions 11 to 13.

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Study Aid

- See Lesson 3.5, Examples 1 to 4.
- Try Chapter Review Questions 16 to 18.

Q: What is conditional probability, and how can you determine it?

A: Conditional probability occurs when two events are dependent. It means that you must consider the probability of A to determine the probability of B. This is written as $P(B \mid A)$. To determine a conditional probability, use the formula below:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}$$

To determine the probability that two dependent events will occur, rearrange the formula as follows:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

For example, Avery draws two cards from a standard deck of 52 playing cards, without replacing the first card. To determine the probability that both cards are hearts, let H represent the event "first card is a heart" and let R represent the event "second card is a heart":

$$P(H \cap R) = P(H) \cdot P(R \mid H)$$

$$P(H \cap R) = \frac{13}{52} \cdot \frac{12}{51}$$

$$P(H \cap R) = \frac{156}{2652}$$

The probability that both cards drawn, without replacement, are both hearts is $\frac{156}{2652}$.

Study Aid

- See Lesson 3.6, Examples 1 and 2.
- Try Chapter Review Questions 19 to 21.

Q: What are independent events, and how can you determine the probability that both will occur?

A: Two events are independent if you can determine the probability of one event without considering the probability of the other event. The following formula can be used to determine the probability that two independent events, A and B, will occur:

$$P(A \cap B) = P(A) \cdot P(B)$$

For example, determine the probability of rolling a 5 on a regular six-sided red die and a 3 on a regular six-sided blue die. The number rolled with the red die has no bearing on the number rolled with the blue die, so these events are independent.

$$P(\text{red die is 5 and blue die is 3}) = P(\text{red die is 5}) \cdot P(\text{blue die is 3})$$

$$P(\text{red die is 5 and blue die is 3}) = \frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36}$$

The probability that a 5 is rolled on the red die and a 6 is rolled on the blue die is $\frac{1}{36}$.

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PRACTISING

Lesson 3.1

1. Is each game fair or not fair? Explain.

a) Chloe and Camila each toss two coins. If all four coins land heads or tails, Chloe wins. If two coins are heads and two coins are tails, Camila wins. Otherwise, it is a tie.

b) Cooper and Alyssa take turns rolling three four-sided dice. If a 1 or a 2 is rolled at least once, then Cooper gets a point. If a 3 or a 4 is rolled at least once, then Alyssa gets a point. (Note that both could get a point on the same roll.)

2. In spinning classes, the tension of a bike is said to be 100% when the pedal can no longer be turned. Bob's spinning instructor asks him to turn the tension up to 130%. Determine the probability that Bob can do this. Explain.

Lesson 3.2

3. On National Aboriginal Day (NAD), celebrated on June 21 every year, cultural events and demonstrations take place across Canada. At a traditional Pow Wow held in Edmonton, Alberta, for NAD, 60% of the people in attendance were female. At the conclusion of the Pow Wow, everyone in attendance receives a gift from the hosts. If the first person receiving a gift is chosen at random:

a) Determine the odds in favour of this person being female.

b) Determine the odds against this person being female.

4. The forecaster predicts a 70% probability of rain tomorrow. What are the odds against rain?

5. Serenity is watching Keir surf. The last seven times that Serenity watched Keir, he fell twice.

a) Determine the odds in favour of Keir falling this time.

b) Determine the odds against Keir falling this time.

6. Ariana plays sponge hockey. She has scored 6 goals in 30 shots on goal. She says that the odds against her scoring a goal are 4 : 5. Is she correct? Explain.

7. Averill is planning her university schedule. She is trying to decide between two different mathematics courses. She has been told that the odds against getting an A in the first course are 8 : 3, and the odds in favour of getting an A in the second course are 6 : 11, based on the results from previous years. Averill wants to get as high a mark as possible. Which course should she take?

Lesson 3.3

8. Two people are randomly chosen from a committee of nine people to be president and secretary. Cameron and Wyatt are on the committee. Determine the probability that they will be chosen for these roles.

9. There are 11 students, including Marina and MacKenzie, on the school swim team. The upcoming swim tournament includes a relay race. The coach has decided to choose the four positions on the relay team (first, second, third, and anchor) randomly. Determine the probability that Marina and MacKenzie will be chosen to be on the relay team.

10. Access to a particular online game is password protected. Every player must create a password that consists of three capital letters followed by four digits. For each condition below, determine the probability that a password chosen at random will contain the letters A, R, and T.

a) Repetitions are not allowed in a password.

b) Repetitions are allowed in a password.

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Lesson 3.4

11. For each of the following, state whether the two events are mutually exclusive or not mutually exclusive. Explain your reasoning.

a) Selecting a prime number or selecting an odd number from a set of 15 balls, numbered 1 to 15

b) Rolling a sum of 8 or a sum of 6 with a pair of six-sided dice, numbered 1 to 6

c) Eating a peach or eating an apple

12. A pinochle deck consists of 48 cards: two copies of the 9, 10, jack, queen, king, and ace cards of all four suits in a standard deck. Hunter is about to draw a card at random from a pinochle deck. If he draws a face card or a spade, he will win a point.

a) Use A and B to represent the two favourable events. Then draw a Venn diagram to illustrate A and B.

b) Are A and B mutually exclusive or not mutually exclusive?

c) Determine the probability of drawing a face card or a spade.

13. The probability that Mya will exercise on Sunday is 0.5. The probability that she will go shopping on Sunday is 0.3. The probability that she will do both is 0.2.

a) Use G and S to represent these two events. Draw a Venn diagram to illustrate G and S.

b) Are G and S mutually exclusive or not mutually exclusive?

c) Determine the probability that Mya will do at least one of these activities on Sunday.

14. There are 24 students in Sergio's Grade 5 class. Based on a survey he conducted, he knows that 19 of these students can swim and 8 can ski. Create a probability problem using Sergio's data. Then solve your problem, using a Venn diagram or a tree diagram.

15. Collect data on a topic that interests you. Create a probability problem using this data. Then solve your problem, using a Venn diagram or a tree diagram.

Lesson 3.5

16. Parker has eight identical black socks and 10 identical white socks loose in his drawer. He pulls out two socks at random. Determine the probability that he pulls out a pair of mismatched socks; that is, one is black and the other is white.

17. The probability that a plane will leave Winnipeg on time is 0.70. The probability that a plane will leave Winnipeg on time and arrive in Calgary on time is 0.56. Determine the probability that a plane will arrive in Calgary on time, given that it left Winnipeg on time.

18. A deck of 40 cards consists of the ace to the 10 from the four suits. A card is dealt to each of two players. Determine the probability that the first card is red and the second card is black.

Lesson 3.6

19. Peyton goes to the gym four days a week. She does a cardio workout each day, using a stair machine or a running machine or taking a salsa class. She also does a strength workout each day, using a weight machine or taking a body sculpt class. Peyton randomly chooses which cardio workout and which strength workout to do each day. Determine the probability that she will use a stair machine and take a body sculpt class the next day she goes to the gym.

20. Suppose that $P(A) = 0.5$, $P(B) = 0.6$, and $P(A \cap B) = 0.3$. Are events A and B independent? Explain.

21. Tanya estimates that her probability of passing French is 0.7 and her probability of passing chemistry is 0.6. Determine the probability that Tanya will:

a) Pass both French and chemistry

b) Pass French but fail chemistry

c) Fail both French and chemistry

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3 Chapter Task

Games and Probability

You can sometimes use probability to increase your chances of winning a game.

For example, in a card game called Gin Rummy, you win by grouping your cards in runs (three or more cards of the same suit in consecutive order) or sets (three or four cards of the same rank). For example, the card hand shown has two sets. If the player is able to get the 4 and 6 of diamonds, the hand will also contain a run.

By carefully watching and remembering the cards that your opponent collects or discards during play, you can often determine your probability of getting a particular card that you want.

? How can you apply your knowledge of probability to increase your odds of winning a game?

A. Choose a game, such as Gin Rummy or cribbage, that has many options and variables.

B. Play the game with a friend. As you play, keep track of situations in which you used probability to decide what card to play. These situations should include, where possible, probabilities or odds, and independent, dependent, or mutually exclusive events.

C. Describe these situations mathematically, indicating the type of probability involved. Explain how you used probability to help you make decisions.

D. Present your results. Your presentation should include at least two helpful strategies to improve a player's chance of winning the game.

Task *Checklist*

- ✔ Did you describe each situation and event clearly?
- ✔ Did you justify the decisions you made?
- ✔ Did you use appropriate mathematical language?

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3 Project Connection

Creating Your Research Question or Statement

A well-written research question or statement clarifies exactly what your project is designed to do. It should have the following characteristics:

- The research *topic* is easily identifiable.
- The *purpose* of the research is clear.
- The question/statement is *focused*. The people who are listening to or reading the question/statement will know what you are going to be researching.

A good question requires thought and planning. Below are three examples of initial questions or statements and how they were improved.

Unacceptable Question/Statement	Why?	Acceptable Question/Statement
Is the Internet changing the way we live?	Too general	How have social networking websites changed the way we interact with each other?
Viable farmland is a shared resource.	Too general	When urban communities spread and development of farmland occurs, the impact is shared.
Are teenagers in my town more conscious about environmental issues than adults?	Too specific, unless you're going to generate your own data.	Are North American teenagers more conscious about environmental issues than adults?

Evaluating Your Research Question or Statement

You can use the following checklist to determine if your research question/statement is effective.

1. Does the question/statement clearly identify the main objective of the research? After you read the question/statement to a few classmates, can they tell you what you will be researching?
2. Are you confident that the question/statement will lead you to sufficient data to reach a conclusion?
3. Is the question/statement interesting? Does it make you want to learn more?
4. Is the topic you chose purely factual, or are you likely to encounter an issue, with different points of view?

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Project Example — Writing a research question

Walther chose pests and invasive species affecting the Western provinces and territories as his topic. Below, he describes how he determined that his research question for this topic is effective.

Walther's Question

My question is, "Which pests or invasive species cause the greatest damage to our natural resources in the Western provinces and territories, and what can be done to control these threats?"

I evaluated my question using the research question checklist, and I feel that it is a good one. Here is why:

1. My question tells what I plan to do—I read my question to members of my family, and they all described what I had in mind.
2. I have already found a lot of data available on pests and invasive species that exist across Canada, and it includes a lot of historical information about their spread and the damage that they cause.
3. I'm really interested in the environment and what we can do to protect it, but I don't know much about this type of problem. I'll find out new things and perhaps some course of action that I can get involved in. Whatever I discover should also interest some of my classmates, as it's about the resources we use in the area where we live.
4. I expect that I will find several different threats that differ by region, and I hope I will be able to conclude what can be done to help in these situations.

Your Turn

A. Write a research question for your topic.

B. Use the checklist to evaluate your question. Adjust your question as needed.

C. Make an appointment to discuss the pros and cons of your research question with your teacher. Be prepared to discuss your plan for collecting the data you will need to come to a conclusion. Adjust your question as needed.

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1-3 Cumulative Review

1. **a)** Represent these sets in a Venn diagram.
- U = {the days of the week}
- A = {the days you go to school}
- B = {the days of the weekend}
- C = {the days of the week that begin with the letter T}

b) List a pair of disjoint subsets in part a), if any.

c) Is each statement true or false? Explain.

i) $A \subset B$ **iii)** $A' = B$

ii) $C \subset A$ **iv)** $n(A) + n(B) + n(C) = n(U)$

2. Erynn surveyed 35 students about their favourite fruit. She recorded her results.

Favourite Fruit	Number of Students
oranges	14
apples	26
neither oranges nor apples	8

a) Draw and label a Venn diagram to represent the data.

b) Determine how many students like oranges or apples.

c) Determine how many students like only oranges or only apples.

3. Describe two sets A and B to which each of the following formulas would apply.

a) $n(A \cup B) = n(A) + n(B)$

b) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

4. Le Centre de développement musical, La Girandole, and L'UniThéâtre offer a summer camp every year in Edmonton. At the Camp Multi-Arts, students can receive instruction in at least one of the following: singing, dancing, and acting. Of the 56 campers:
- 35 were instructed in singing, 38 in dancing, and 33 in acting.
- 21 were instructed in singing and dancing.
- 18 were instructed in singing and acting.
- 23 were instructed in acting and dancing.

How many campers received instruction in all three art forms?

5. Guido is planning a summer trip to visit ancient ruins in southern Italy. Give four search terms Guido might use to find information on this topic, and use a Venn diagram to illustrate how using quotation marks and the "&" symbol can help him refine his search.

6. You roll a regular six-sided die and flip a coin.

a) Draw a tree diagram to show all possible outcomes.

b) Verify that you have listed all possible outcomes using the Fundamental Counting Principle.

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7. The camera club has 15 members. In how many ways can:
- **a)** All the members be arranged in a row for a yearbook photo?
- **b)** The club choose a president, vice-president, and treasurer?
- **c)** Jill and two other members be chosen to take pictures of the next football game?

8. A softball team has a record of 13 wins and 5 losses. How many different ways could this record have occurred?

9. Solve for n:
$_nC_4 = {}_nP_3$

10. There are 7 parents and 9 students who volunteer to be on a school fundraising committee. How many different six-person committees are possible if the committee must consist of:
a) 3 parents and 3 students? **b)** At least 1 student?

11. Six students are running for student council president. Suppose that each student is equally likely to win the election. What is the probability that Sonya will finish in the top three on election day?

12. In the game of bridge, four players are dealt 13 cards from a well-shuffled standard deck of 52 playing cards. Determine the probability that a player:
a) Is dealt all the hearts. **b)** Is dealt all the kings.

13. Enzo rolls two regular six-sided dice. Determine the probability of each event.
- **a)** The sum is 3 or 12.
- **b)** The sum is odd, or the sum is greater than 7.
- **c)** The sum is even, given that the first die is 1.
- **d)** The first die is less than 3, and the second die is greater than 5.

14. Jan and Stan are evenly matched table tennis players. However, each time Jan loses a game, her probability of winning the next game decreases by $\frac{1}{4}$. When she wins, her probability of winning the next game increases by $\frac{1}{8}$. If they play two games, determine the probability that:
- **a)** Jan wins at least one game.
- **b)** Stan wins both games.

15. A euchre deck consists of the 9, 10, jack, queen, king, and ace of all four suits. This results in a euchre deck of 24 cards. Audrey draws two cards from a well-shuffled euchre deck. Determine the probability that both cards are queens:
- **a)** If the first card drawn is replaced.
- **b)** If the first card drawn is not replaced.

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